
2016

Set: I

Question: 1 – 26

ii - xxiv

Section: A

Question: 1

What is the amount of work done in moving a point charge Q around a circular arc of radius ' r ' at the center of which another point charge ' q ' is located ? [1]

Answer:

Potential difference between any two points on circumference of circle is zero. So work done ($w = qV$) will be zero.

Question: 2

Define mobility of a charge carrier. What is its relation with relaxation time. [1]

Answer:

It is defined as drift velocity per unit electric field

$$\mu = \frac{V_d}{E} = \frac{eE\tau}{mE} = \frac{e\tau}{m}$$

Question: 3

What can be the cause of helical motion of a charged particle? [1]

Answer:

When there is an angle between velocity of charge particle and magnetic field, then the vertical component of velocity ($V \sin \theta$) will rotate the charge particle on circular path, but horizontal component ($V \cos \theta$) will move the charge particle in straight line. So path of the charge particle becomes helical.

Question: 4

Why can't we see clearly through fog? Name the phenomenon responsible for it. [1]

Answer:

When light falls on fog then scattering takes place so the particles of fog become visible and light is scattered by the fog and will not reach the object.

Question: 5

A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. What are the frequencies of the side bands produced? [1]

Answer:

$$\begin{aligned} \text{Maximum frequency} &= 5\text{ kHz} + 2\text{ MHz} = 5 \times 10^3 + 2 \times 10^6 = 10^3 (5 + 2000) \\ &= 2005 \times 10^3 = 2.005 \times 10^6 \text{ Hz} \end{aligned}$$

$$\begin{aligned} \text{Minimum frequency} &= 2\text{ MHz} - 5\text{ kHz} = 2 \times 10^6 - 5 \times 10^3 = (2000 - 5) \times 10^3 \\ &= 1995 \times 10^3 = 1.995 \times 10^6 \text{ Hz} \end{aligned}$$



Section: B

Question: 6

When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m/s. If the electron density in the wire is $8 \times 10^{28} \text{ m}^{-3}$, calculate the resistivity of the material of wire. [2]

Answer:

We know $I = neAv_d$ and $I = \frac{V}{R}$

$$\text{So } \frac{V}{R} = neAv_d$$

$$\frac{V}{nev_d \ell} = \frac{RA}{\ell} \Rightarrow \rho = \frac{V}{nev_d \ell}$$

$$\begin{aligned} \rho &= \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1} \\ &= 1.5625 \times 10^{-5} \Omega \text{ m} \end{aligned}$$

Question: 7

A proton and an α particle are accelerated through the same potential difference. Which one of the two has (i) greater de-Broglie wavelength, and (ii) less kinetic energy? Justify answer. [2]

Answer:

i. We know $\lambda_\alpha = \frac{0.1012}{\sqrt{V}} \text{ \AA}$, $\lambda_p = \frac{0.2863}{\sqrt{V}} \text{ \AA}$
 λ will be greater for porton for same V

ii. $\therefore \text{K.E.} = qV$
So, $(\text{K.E.})_p = eV$
And, $(\text{K.E.})_\alpha = 2eV$
K.E. of proton will be less.

Question: 8

When is H_α line in the emission spectrum of hydrogen atom obtained? Calculate the frequency of the photon emitted during this transition. [2]

Answer:

When electron move from $n = 2$ energy level to $n = 1$ energy level then we obtain H_α line in H-spectrum.

$$\begin{aligned} E &= E_{n_2} - E_{n_1} \\ &= \frac{-13.6}{2^2} + \frac{+13.6}{1^2} \\ &= \frac{-13.6}{4} + \frac{13.6}{1} \\ &= 13.6 \left[1 - \frac{1}{4} \right] \\ &= \frac{3}{4} (13.6) \end{aligned}$$



$$\begin{aligned}
 E &= 10.2 \text{ eV} \\
 \text{we know } E &= h\nu \\
 h\nu &= 10.2 \times 1.6 \times 10^{-19} \\
 \nu &= \frac{10.2 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} \\
 &= \frac{10.2 \times 1.6 \times 10^{15}}{6.62} \\
 &= \frac{16.2}{6.62} \times 10^{15} \\
 &= 2.46 \times 10^{15} \text{ Hz}
 \end{aligned}$$

OR

Calculate the wavelength of radiation emitted when electron in a hydrogen atom jumps from $n = \infty$ to $n = 1$

Answer:

$$\begin{aligned}
 \text{We know, } \frac{1}{\lambda} &= R_h \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \\
 \frac{1}{\lambda} &= R_h \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right] \\
 \frac{1}{\lambda} &= R_h \\
 \lambda &= \frac{1}{R_h} \\
 &= 911.6 \times 10^{10} \text{ m}
 \end{aligned}$$

Question: 9

Why is base band signal not transmitted directly? Give any two reasons.

[2]

Answer:

Base band signal not transmitted directly because

- Due to higher wavelength (low frequency) antenna or aerial size required for transmission will be very high.
- For linear antenna (length l), the power radiated is proportional to $\left(\frac{l}{\lambda}\right)^2$ hence the effective radiated by a long wavelength baseband signal would be small.

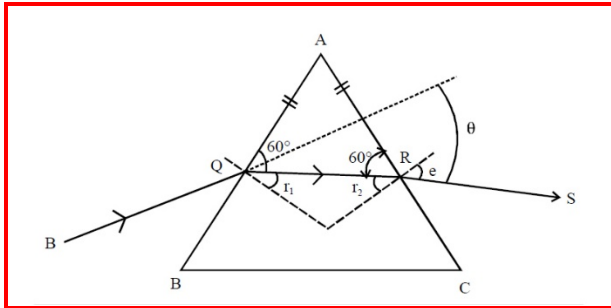
Question: 10

A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS such that $AQ=AR$. If the angle of prism $A=60^\circ$ and refractive index of material of prism is 3, calculate angle θ .

[2]

Answer:





$\therefore \angle A = 60^\circ$, but $AQ = AR$

So, $\angle AQR = \angle ARQ = 60^\circ$

and, $i = e$ Now

$$\mu = \frac{\sin i}{\sin r_1}$$

$$\sqrt{3} = \frac{\sin i}{\sin 30^\circ}$$

$$\text{or, } \sin i = \frac{\sqrt{3}}{2} = 60^\circ$$

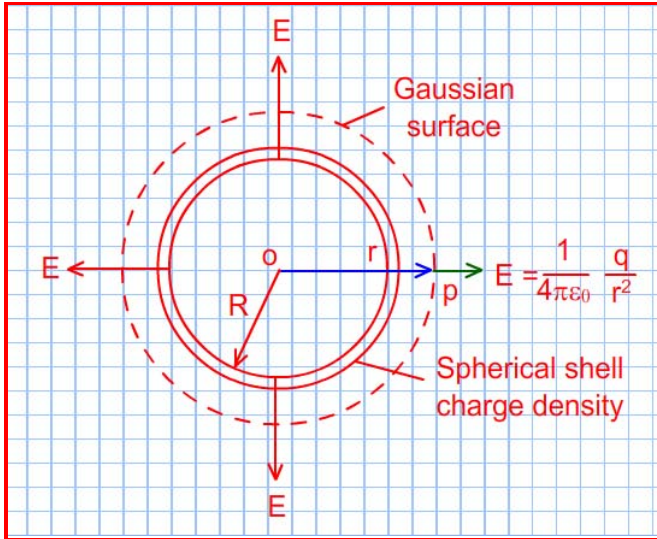


Section: C

Question: 11

Find the electric field intensity due to a uniformly charged spherical shell at a point (i) outside the shell and (ii) inside the shell. Plot the graph of electric field with distance from the center of the shell. [3]

Answer:



When point P lies outside the spherical shell →
Then, Flux through the Gaussian surface,

$$\phi_E = E \times 4\pi r^2 \text{ and}$$

By Gauss's theorem

$$\phi_E = \frac{q}{\epsilon_0}$$

$$\therefore E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\text{or, } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

When point P lies inside the spherical shell → then charge enclosed by the Gaussian surface is zero,

i.e. $q = 0$ Flux through the Gaussian surface

$$\phi_E = E \times 4\pi r^2$$

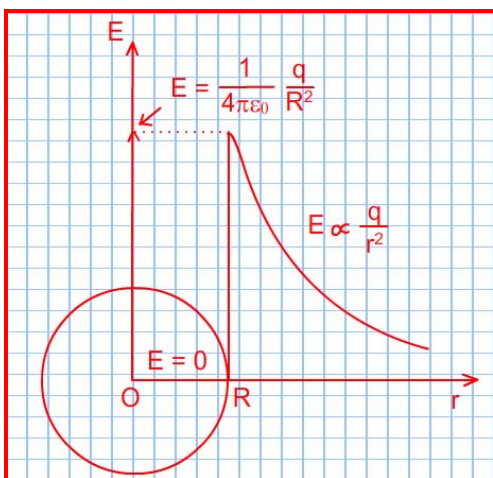
$$\text{Applying Gauss's theorem } \phi_E = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = 0$$

$$\text{or, } E = 0 \text{ [For } r < R]$$

Hence electric field due to a uniformly charged spherical shell is zero at all points inside the shell.





Question: 12

Two identical cells of emf 1.5 V each joined in parallel supply energy to an external circuit consisting of two resistances of 7 each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell. [3]

Answer:

$$\varepsilon = 1.5 \text{ V } V = 1.4 \text{ V}$$

$$\text{Total } R = \frac{7 \times 7}{7 + 7} = \frac{49}{14} = 3.5 \Omega$$

Let r' is total internal resistance then

$$r' = \left(\frac{\varepsilon - V}{V} \right) R = \left(\frac{1.5 - 1.4}{1.4} \right) \times 3.5 = 0.25 \Omega$$

So, Internal resistance of each cell

$$r' = \frac{r}{2}$$

$$\begin{aligned} \text{or, } r &= 2r' \\ &= 0.25 \times 2 \\ &= 0.5 \Omega \end{aligned}$$

Question: 13

State Ampere's circuital law. Use this law to find magnetic field due to straight infinite current carrying wire. How are the magnetic field lines different from the electrostatic field lines? [3]

Answer:

Ampere's circuital law \rightarrow states that the line integral of the magnetic field \vec{B} around any closed circuit is equal to μ_0 (permeability constant) times the total current I threading or passing through this closed circuit. Mathematically,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

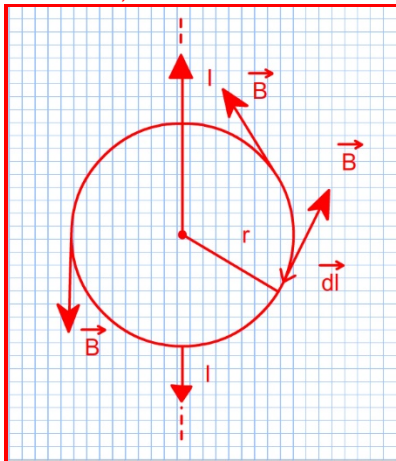
Magnetic field due to straight infinite current carrying wire \rightarrow

Figure shows a circular loop of radius r around an infinitely long straight wire carrying current I .

As the field lines are circular, the field \vec{B} at any point of the circular loop is directed along the tangent to the circle at that point. By symmetry, the magnitude of field \vec{B} is same at every point of the circular loop.



Therefore,



$$\oint \vec{B} \cdot d\vec{l} = \oint B dl \cos 0^\circ = B \oint dl = B \cdot 2\pi r$$

From Ampere's circuital law

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

OR

State the principle of a cyclotron. Show that the time period of revolution of particles in a cyclotron is independent of their speeds. Why is this property necessary for the operation of a cyclotron?

Answer:

Principle of cyclotron → A charged particle can be accelerated to very high energies by making it pass through a moderate electric field a number of times. This can be done with the help of a perpendicular magnetic field which throws the charged particle into a circular motion, the frequency of which does not depend on the speed of the particle and the radius of the circular orbit.

Time period of revolution → Let a particle of charge q and mass m enter a region of magnetic field \vec{B} with a velocity \vec{v} , normal to the field \vec{B} . The particle follows a circular path, the necessary centripetal force being provided by the magnetic field. Therefore,
Magnetic force on charge q = Centripetal force on charge q

$$\text{or, } q v B \sin 90^\circ = \frac{mv^2}{r}$$

$$\text{or, } r = \frac{mv}{qB}$$

$$\text{Period of revolution of the charged particle is given by } T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Clearly Time period is independent on the velocity of the particle.

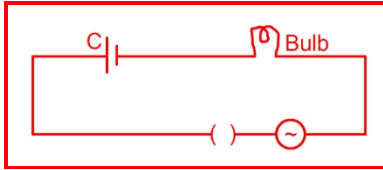
It is necessary for the operation of cyclotron so that frequency of revolution must be equal to the frequency of AC source.

Question: 14



- i. When an AC source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero. [1½]

Answer:



We know $P_{av} = E_{rms} I_{rms} \cos \phi$

For capacitive circuit phase angle between current and voltage is $\frac{\pi}{2}$.

$$\text{So } P_{av} = E_{rms} I_{rms} \cos \frac{\pi}{2}$$

$$P_{av} = 0$$

- ii. A bulb is connected in series with a variable capacitor and an A.C source as shown. What happens to the brightness of the bulb when the key is plugged in and capacitance of the capacitor is gradually reduced? [1½]

Answer:

Reducing the capacitance implies $\left(X_c = \frac{1}{\omega C} \right)$ increase in X_c and consequently current decrease in circuit so the glow of the bulb decrease.

Question: 15

How are electromagnetic waves produced? What is the source of energy of these waves? Write mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis. Write any two important properties of electromagnetic wave. [3]

Answer:

EM wave produced by oscillating charged particle.

Mathematical expression for electromagnetic wave travel along z – axis →

$$E_x = E_0 \sin (Kz \cdot wt) \text{ and } B_y = B_0 \sin (Kz \cdot wt)$$

Properties of EM waves→

- They are produced by accelerate charge particle.
- They does not required medium for their propagation.

Question: 16

- i. Derive Snell's law on the basis of Huygen's wave theory when light is travelling from a denser to a rarer medium. [1½]

Answer:





First of all, the disturbance from wave front AB strikes at the point B. By the time $\left(t = \frac{AA'}{C_1}\right)$ disturbance from A reaches A', disturbance from B would have spread in the second medium in the form of hemispherical wavelet of radius $BB' \left(= C_2 t = C_2 \times \frac{AA'}{C_1}\right)$. Tangent from A' on this wavelet gives refracted wave front A'B'.

$$\text{In } \triangle BB'A', \sin r = \frac{BB'}{BA'}$$

$$\frac{\sin i}{\sin r} = \frac{\frac{AA'}{BA'}}{\frac{BB'}{BA'}} = \frac{AA'}{BB'} = \frac{C_1 t}{C_2 t} = \frac{C_1}{C_2} = \text{constant}$$

This is Snell's law or first law of refraction.

ii. Draw the sketches to differentiate between plane wavefront and spherical wavefront. $\left[1\frac{1}{2}\right]$

	
Spherical wave front	Plane wave front



Question: 17

[2]

State two important properties of photon which are used to write Einstein's photoelectric equation. Define

- Stopping potential
- Threshold frequency

Using Einstein's equation and drawing necessary plot between relevant quantities.

Answer:

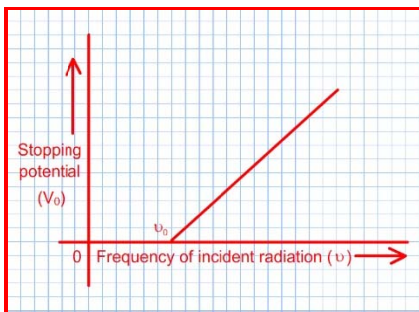
Properties of photon→

- The rest mass of Photon is 0
- Energy of Photon is given by $E = h\nu = \frac{hc}{\lambda}$

Stopping Potential→

The value of negative potential at which. Photoelectric emission stop is called stopping Potential.

Threshold freq_n→ The minimum freq_n. Required to eject the electron from metal surface is called threshold frequency.

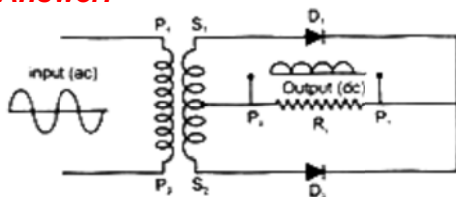
**Question: 18**

- Name two important processes that occur during the formation of a pn junction. [1½]

Answer:

Two important processes occur during the formation of a p-n junction are diffusion and drift.

- Draw the circuit diagram of a full wave rectifier along with the input and output waveforms. Briefly explain how the output voltage/current is unidirectional. [1½]

Answer:

The ac to be transformed is connected as shown across primary P₁ P₂ of a transformer.

In 1st half of ac cycle, suppose P₁ is negative and P₂ is positive. This make S₁ positive and S₂ negative. As a result, D₁ is forward biased and hence conducting. The current flows through D₁



and from P_1 to P_2 through R_1 and we get an output. D_2 does not conduct in this half as it is reverse biased.

Similarly in second half, D_1 does not conduct but D_2 conducts being forward biased making the current flow from P_1 to P_2 again.

Question: 19

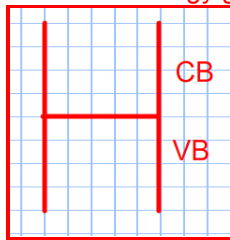
- i. Distinguish between a conductor and a semiconductor on the basis of energy band diagram.

[1 $\frac{1}{2}$]

Answer:

Conductor→

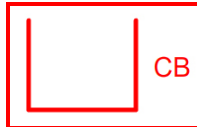
- a. In conductor valance band is partially filled (eg. Li, K, Na) or completely filled. (eg. B, Mg, Zn)
- b. Forbidden energy gap between conduction band and valance band is 0.



c.

Insulator→

- a. In insulator valance band is completely filled and conduction band is empty.
- b. Forbidden energy gap between conduction band and valance band is 3eV.

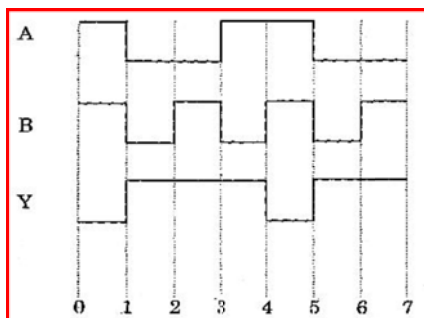


c. $\Delta E_g \geq 3\text{eV}$



- ii. The following figure shows the input waveforms (A, B) and the output waveform (Y) of a gate. Identify the gate, write its truth table and draw its logic symbol.

[1 $\frac{1}{2}$]



Answer:
gate is NAND gate

Truth Table

A	B	AB	$Y = \overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Question: 20

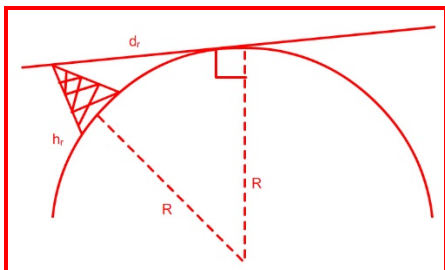
[3]

What is space wave propagation? State the factors which limit its range of propagation. Derive an expression for the maximum line of sight distance between two antennas for space wave propagation.

Answer:

Space wave travels in a straight line from transmitting antenna to the receiving antenna. Range of propagation will be limited due to following factors.

- At high frequency waves scatter more easily.
- There should not be any part of earth lies between transmitter and receiver.



From Figure

$$(h_T + R)^2 = R^2 + d_T^2$$

$$R^2 + 2Rh_T + h_T^2 = R^2 + d_T^2$$

$$\text{But } h_T^2 \ll 2Rh_T$$

$$\text{So } d_T^2 = 2Rh_T$$

$$\Rightarrow d_T = \sqrt{2Rh_T}$$

$$\text{Similarly } d_R = \sqrt{2Rh_R}$$

$$\text{So } d_m = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Question: 21

- Derive the mathematical expression for law of radioactive decay for a sample of a radioactive nucleus.

[1½]

Answer:



The number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the sample. If N is the number of nuclei in the sample and ΔN undergo decay in time.

$$\Delta t \text{ then } \frac{\Delta N}{\Delta t} \propto N$$

$$\text{or, } \frac{\Delta N}{\Delta t} = -\lambda N,$$

where λ is called the radioactive decay constant or disintegration constant. (in the limit $\Delta t \rightarrow 0$)

$$\frac{dN}{dt} = -\lambda N$$

$$\text{or, } \frac{dN}{N} = -\lambda dt$$

Now, integrating both sides of the above equation, we get,

$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\text{or, } \ln N - \ln N_0 = -\lambda t$$

- ii. How is the mean life of a given radioactive nucleus related to the decay constant? $[1\frac{1}{2}]$

Answer:

$$\tau = \frac{\lambda N_0 \int_0^{\infty} t e^{-\lambda t} dt}{N_0}$$

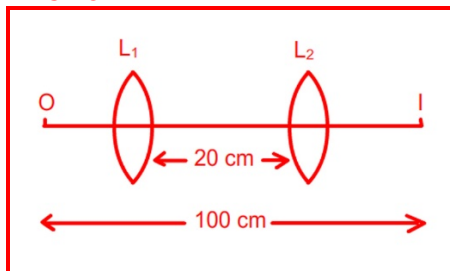
$$= \lambda \int_0^{\infty} t e^{-\lambda t} dt$$

One can show by performing this integral that $\tau = 1/\lambda$

Question: 22

- i. A screen is placed at a distance of 100 cm from an object. The image of the object is formed on the screen by a convex lens for two different locations of the lens separated by 20 cm. Calculate the focal length of the lens used. $[1\frac{1}{2}]$

Answer:



$$\frac{1}{f} = \frac{1}{(100-u)} + \frac{1}{u} \quad \rightarrow (1)$$

$$\frac{1}{f} = \frac{1}{(80-u)} + \frac{1}{u+20} \quad \rightarrow (2)$$

on solving equation (1) & (2) $u = 40$ cm

So, $v = 100 - 40 = 60$ cm.



Now, from len's formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{40} = \frac{4+6}{240}$$

$f = 24$ cm.

- ii. A converging lens is kept coaxially in contact with a diverging lens - both the lenses being of equal focal length. What is the focal length of the combination?

$[1\frac{1}{2}]$

Answer:

Let focal length of two lens are f then from formula.

$$\frac{1}{f_{\text{equ}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_{\text{equ}}} = \frac{1}{f} - \frac{1}{f}$$

$$\text{So } \frac{1}{f_{\text{equ}}} = \infty$$



Section: D

Question: 23

[4]

Seema's uncle was advised by his doctor to have an MRI (Magnetic Resonance Imaging) scan of his brain. Her uncle felt it to be expensive and wanted to postpone it.

When Seema learnt about this, she took the help of her family and also approached the doctor, who also offered a substantial discount. She then convinced her uncle to undergo the test to enable the doctor to know the condition of his brain. The information thus obtained greatly helped the doctor to treat him properly.

Based on the above paragraph, answer the following questions:

- a. What according to you are the values displayed by Seema, her family and the doctor?

Answer:

Presence of mind. High degree of general awareness. Ability to take prompt decisions. Concern for his uncle.

- b. What could be the possible reason for MRI test to be so expensive?

Answer:

For MRI a conducting material have the unique property (Super conductor) required. Which is Very expensive.

- c. Assuming that MRI test was performed using a magnetic field of 0.1 T., find the minimum and maximum values of the force that the magnetic field could exert on a proton (charge = 1.6×10^{-19} C) moving with a speed of 104 m/s.

Answer:

Maximum force = $qvB_{\max} \sin 90^\circ$ (when $\vec{v} \perp \vec{B}$)

$$= (1.6 \times 10^{-19}) (10^4) (1)$$

$$= 1.6 \times 10^{-15} \text{ N}$$

Minimum force = Zero

When the proton moves parallel or antiparallel to the magnetic field direction.



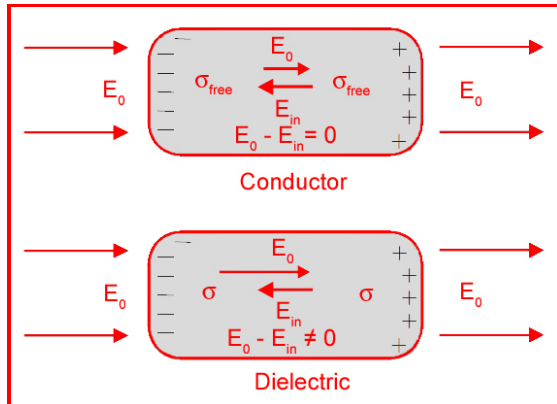
Section: E

Question: 24

[5]

- a. Distinguish, with the help of a suitable diagram, the difference in the behavior of a conductor and a dielectric placed in an external electric field. How does polarized dielectric modify the original external field?

Answer:



When Polarized dielectric slab is placed in external electric field (E_0) then due polarization. Polarized electric field (E_p) inside the dielectric slab in the opposite direction of that of external field. So net electric field inside the dielectric slab decrease.

- b. A capacitor of capacitance C is charged fully by connecting it to a battery of emf E . It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change?
- Charge stored by the capacitor.
 - Field strength between the plates.
 - Energy stored by the capacitor.

Justify your answer in each case.

Answer:

- Charge stored by the capacitor remain same because charge is conserve.
- Field strength remain same because $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$ and it does not depends on the distance between the plate.
- Energy stored by the capacitor is double because on increasing distance betⁿ the plate two times capacitance between the plates reduced to half, so energy stored $\left(\frac{Q^2}{2c}\right)$ increase two times.

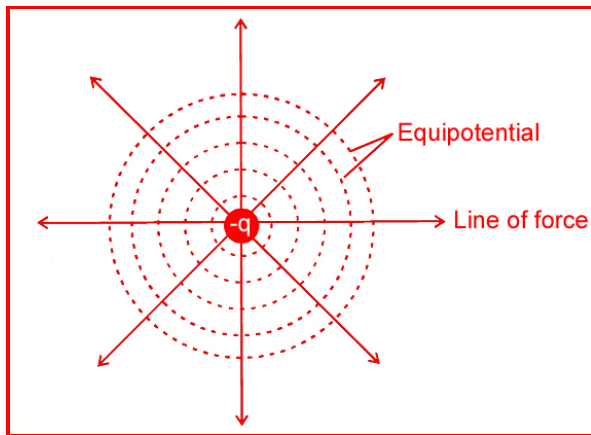
OR

- a. Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point. Draw a sketch of equipotential surfaces due to a single charge ($-q$), depicting the electric field lines due to the charge.

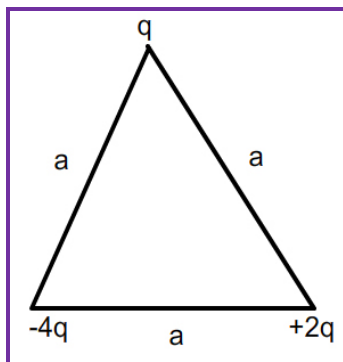
Answer:



Because if electric line of force is not perpendicular to the equipotential surface then there are two component of electric field. The component which is parallel to the surface set up electric current in equilibrium state. Which is not possible for negative charge equipotential surface are as



- b. Obtain an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.



Answer:

$$\begin{aligned}
 W &= \frac{Kq(-4q)}{a} + \frac{Kq \times 2q}{a} + \frac{K \times (-4q)2q}{a} \\
 &= -\frac{4Kq^2}{a} + \frac{2Kq^2}{a} - \frac{8Kq^2}{a} \\
 &= -\frac{10Kq^2}{a}
 \end{aligned}$$

So, work done to dissociate the system of three charges is $w = -\frac{10Kq^2}{a}$

Question: 25

[5]

- a. When a bar magnet is pushed towards (or away) from the coil connected to a galvanometer, the pointer in the galvanometer deflects. Identify the phenomenon causing this deflection and write the factors on which the amount and direction of the deflection depends. State the laws describing this phenomenon.

Answer:

Electromagnetic Induction.

Factors on which strength of induced electric current and direction depends.



- i. Speed of magnet
- ii. Polarity

Faraday's Law of Induction

First Law →

Whenever magnetic flux linked with the coil change then emf induced in it.

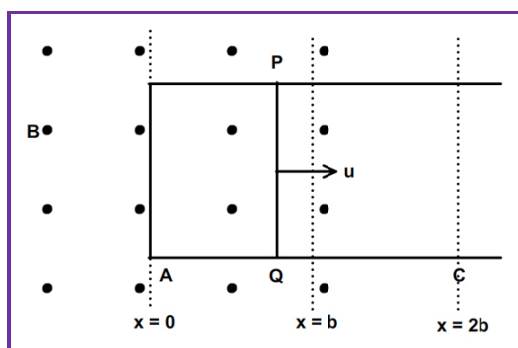
Second Law→

The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

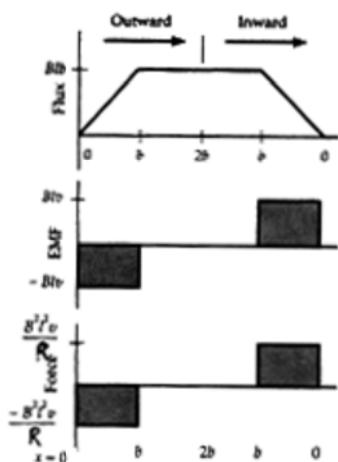
Lenz's Law

The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

- b. Sketch the change in flux, emf and force when a conducting rod PQ of resistance R and length moves freely to and fro between A and C with speed v on a rectangular conductor placed in uniform magnetic field as shown in the figure.



Answer:



We first consider the forward motion from $x = 0$ to $x = 2b$.

Flux, $\phi = Bx$ for $0 \leq x < b$
 $= Bb$ for $b \leq x < 2b$

Induced emf, $\varepsilon = \frac{d\phi}{dt}$

$$\therefore \varepsilon = \frac{d}{dt} (Bx)$$



$$\begin{aligned}
 &= -B \frac{dx}{dt} \\
 &= -B/v \text{ for } 0 \leq x < b \\
 \text{and } \varepsilon &= \frac{d}{dt} (B/b) = 0 \text{ for } b \leq x < 2b
 \end{aligned}$$

When the induced emf is non-zero, the magnitude of the induced current is $I = \frac{B/v}{R}$ [R= resistance of conductor PQ]

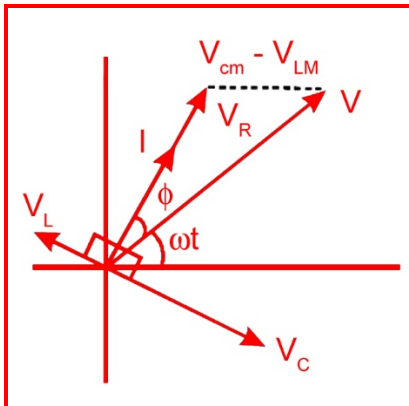
The force required to keep the arm PQ in constant motion is lB . Its direction is to the left. In magnitude,

$$\begin{aligned}
 F &= \frac{B^2 l^2 v}{R} \text{ For } b \leq x < b \\
 &= 0 \text{ For } b \leq x < 2b
 \end{aligned}$$

OR

In a series LCR circuit connected to an a.c. source of voltage $v = v_m \sin \omega t$, use phasor diagram to derive an expression for the current in the circuit. Hence obtain the expression for the power dissipated in the circuit, Show that power dissipated at resonance is maximum.

Answer:



The phasor relation whose vertical component gives the above equation is $V_L + V_R + V_C = V$

This relation is represented in figure. Since V_C and V_L are always along the same line and in opposite directions.

$$V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$$

Substituting the values of V_{Rm} , V_{Cm} and V_{Lm} from Eq. into the above equation. We have

$$V_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$i_m^2 [R^2 + (X_C - X_L)^2]$$

$$\text{or, } i_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

By analogy to the resistance in a circuit, we introduce the impedance Z in an A C circuit:



$$i_m = \frac{V_m}{Z}$$

$$\text{where } Z = \sqrt{R^2 + (V_C - V_L)^2}$$

Power In LCR Circuit→

The instantaneous power p supplied by the source is $p = vi = (v_m \sin \omega t) \times [i_m \sin(\omega t + \phi)]$

$$= \frac{V_m i_m}{2} [\cos \phi + \cos(2\omega t + \phi)]$$

The average power over a cycle is given by the average of the two terms in R.H.S. of Eq. It is only the second term which is time-dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore

$$P = \frac{V_m i_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos \phi$$

$$= V I \cos \phi$$

This can also be written as, $P = I^2 Z \cos \phi$

Power dissipated at resonance in LCR circuit : At resonance $X_C - X_L = 0$. and $\phi = 0$. Therefore.

$\cos \phi = 1$ and $P = I^2 Z = I^2 R$. That is, maximum power is dissipated in a circuit (through R) at resonance.

Question: 26

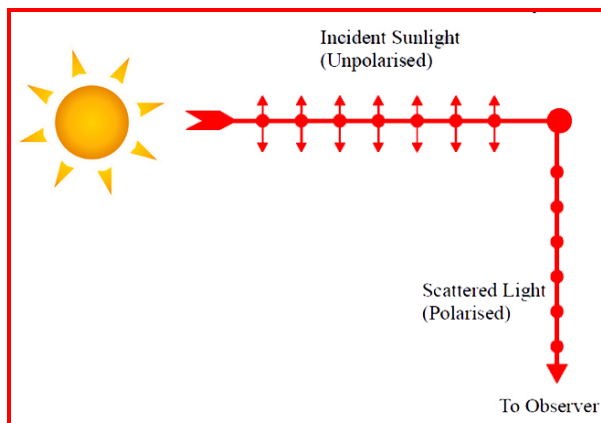
- a. Why does unpolarised light from a source show a variation in intensity when viewed through a polaroid which is rotated? Show with the help of a diagram, how unpolarised light from sun

gets linearly polarized by scattering.

$$\left[2\frac{1}{2}\right]$$

Answer:

A Polaroid consists of long chain molecules. Aligned in a particular direction. The electric vectors along the direction of aligned molecules get absorbed. Thus if the unpolarised light from a source passes through a Polaroid, its intensity is reduced by half. Rotating the polaroid has no effect on the transmitted beam and transmitted. Intensity remains constant.

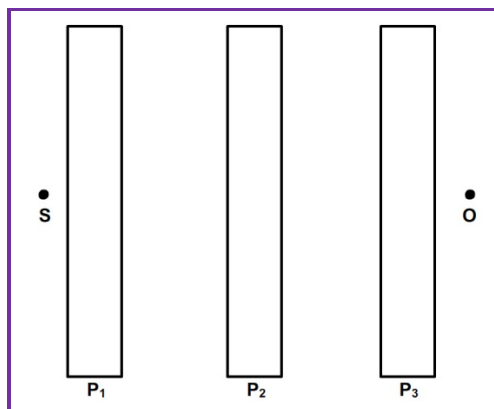


- b. Three identical polaroid sheets P_1 , P_2 and P_3 are oriented so that the pass axis of P_2 and P_3 are inclined at angles of 60° and 90° respectively with the pass axis of P_1 . A monochromatic source S of unpolarized light of intensity I_0 is kept in front of the polaroid sheet P_1 as shown in

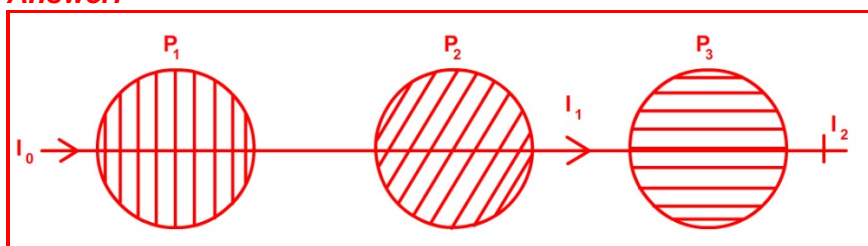


the figure. Determine the intensities of light as observed at O, when polaroid P_3 is rotated with respect to P_2 at angles $\theta = 30^\circ$ and 60° .

$[2\frac{1}{2}]$



Answer:



Intensity of light passes through polaroid P_2

$$I_1 = \frac{I_0}{2} \cos^2 60^\circ$$

$$= \frac{I_0}{2} \times \left(\frac{1}{4}\right)$$

$$= \frac{I_0}{8}$$

when Polaroid (P_3) rotated by 30° angle then intensity of light passes through polaroid P_3

$$I_2 = I_1 \cos^2 90^\circ$$

$$= \frac{I_0}{8} \times 0$$

$$= 0$$

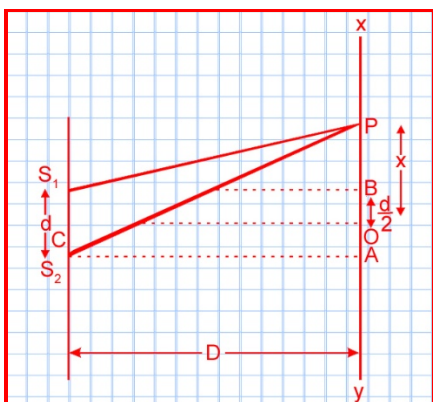
OR

- a. Derive an expression for path difference in Young's double slit experiment and obtain the condition for constructive and destructive interference at a point on the screen.

Answer:

Path difference in YDSE





In figure S_1 and S_2 are two narrow closely spaced slits illuminated by monochromatic light of wavelength λ , XY is the screen on which interference pattern is observed, If S_1 and S_2 emit light in same phase, then for point O, on right bisector of S_1S_2 the path difference receives light in same phase. The superposition at O is constructive producing a bright point, called the central maxima. The intensity at any point P at a distance x from O depends on the path difference between light reaching P from S_1 and S_2 .

We have path difference $p = S_2P - S_1P$

$$= D^2 + (AO + OP)^2$$

$$= D^2 + \left(x + \frac{d}{2}\right)^2$$

$$\text{Similarly } S_1P^2 = D^2 + \left(x - \frac{d}{2}\right)^2$$

$$(S_2P + S_1P)(S_2P - S_1P) = 4 \cdot x \cdot \frac{d}{2} = 2xd$$

If P is very closed to O then $S_2P + S_1P = 2D$ and $S_2P - S_1P = p$

$$\therefore 2D \cdot p = 2xd$$

$$\text{or, } p = \frac{xd}{D}$$

Condition for constructive interference →

The resultant intensity at a point is maximum when the phase difference between the two superposing waves is an even multiple of π or path difference is an integral multiple of wavelength λ .

Condition for destructive interference →

The resultant intensity at a point is minimum when the phase difference between the two superposing waves is an odd multiple of π or the path difference is an odd multiple of wave length $\frac{\lambda}{2}$.

- b. The intensity at the central maxima in Young's double slit experiment is I_0 . Find out the intensity at a point where the path difference is $\frac{\lambda}{6}$, $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$

Answer:



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

When $\phi = 0$, then Intensity is I_0 (i)

$$\text{and, } I_1 = I_2 = I$$

$$\text{So, } I_0 = (\sqrt{I} + \sqrt{I})^2$$

$$I_0 = 4I \Rightarrow I = \frac{I_0}{4} \text{ when path difference is } \frac{\lambda}{6}$$

$$\text{Then } \phi = \frac{\pi}{3}$$

$$\text{So, } I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \frac{\pi}{3}$$

$$= I_1 + I_2 + \sqrt{I_1 I_2}$$

$$\text{But } I_1 = I_2 = \frac{I_0}{4}$$

$$\text{So, } I = \frac{I_0}{4} + \frac{I_0}{4} + \frac{I_0}{4}$$

$$I = \frac{3I_0}{4}$$

When path difference is $\frac{\lambda}{4}$ (ii)

$$\text{Then } \phi = \frac{\pi}{2}$$

$$\text{So, } I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \frac{\pi}{2} = \frac{eE\tau}{mE} = \frac{e\tau}{m}$$

$$= I_1 + I_2$$

$$\text{But, } I_1 = I_2 = \frac{I_0}{4}$$

$$\text{So, } I = \frac{I_0}{4} + \frac{I_0}{4}$$

$$I = \frac{I_0}{2}$$

When path difference is $\frac{\lambda}{3}$ (ii)

$$\text{Then } \phi = \frac{2\pi}{3}$$

$$\text{So, } I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \frac{2\pi}{3}$$

$$= I_1 + I_2 - \sqrt{I_1 I_2}$$

$$\text{But } I_1 = I_2 = \frac{I_0}{4}$$

$$\text{So, } I = \frac{I_0}{4} + \frac{I_0}{4} - \frac{I_0}{4}$$

$$I = \frac{I_0}{4}$$

