2011

Question 1
Define the term 'threshold frequency' in relation to photoelectric effect. [1]

Answer:
Threshold frequency is defined as the minimum frequency of incident radiation which can cause Photoelectric emission. It is different for different metal

Question 2
The peak value of e.m.f in a.c. is E_0. Write its (i) rms and (ii) average value over a complete cycle. [1]

Answer:
E_0 = peak value of emf

1. rsm value $[E_{rms}] = \frac{E_0}{\sqrt{2}}$
2. average value [Eav] = zero

Question 3
Two insulated charged copper spheres A and B of identical size have charges q_A and q_B respectively. A third sphere C of the same size but uncharged is brought in contact with the first and then in contract with the second and finally removed from both. What are the new charges on A and B? [1]

Answer:
New charge on A is $\frac{q_A}{2}$ and new charge on B is $\frac{q_A + 2q_B}{2}$

Question 4
A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the radii of the circular paths described by them? [1]

Answer:
Charge on deutron (qd) = charge on proton (qp)
qd = qp

Radius of circular path (r)= $\frac{p}{B_q}$ $\quad (\because qvB = \frac{mv^2}{r})$

$r \propto \frac{1}{q}$ [for constant momentum (P)]

so, $\frac{r_p}{r_d} = \frac{q_d}{q_p} = \frac{q_p}{q_d} = 1$

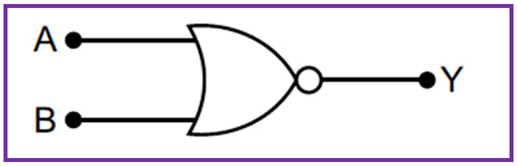
Hence, $r_p : r_d$ = 1:1

Question 5
What is the function of a 'Repeater' in a communication system? [1]

Answer:
A repeater which is a combination of a transmitter, on amplifier and a receiver which picks up signal from the transmitter, amplifies and retransmits it to the receiver.

Question 6
Draw the logic circuit of a NAND gate and write its truth table. [1]

Answer:
Logic circuit of NAND gate:



Input		Output
A	**B**	**Y**
0	0	1
0	1	1
1	0	1
1	1	0

Question 7
How is the mean life of a radioactive sample related to its half-life? [1]

Answer:
Mean life (τ) and half-life ($T_{\frac{1}{2}}$) are related as:

$$\tau = \frac{T_{\frac{1}{2}}}{0.6931}$$

Question 8
Write two uses of microwaves. [1]

Answer:
Uses of microwaves:

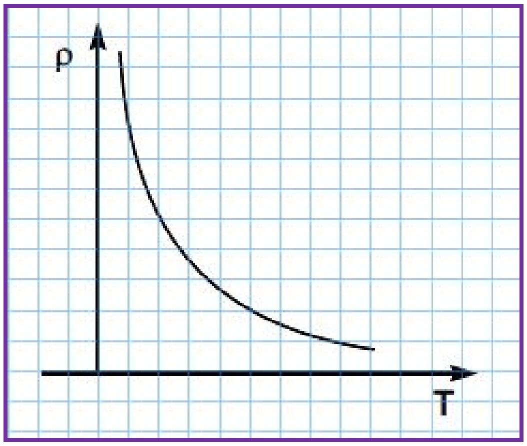
i. In long distance communications.
ii. In radar

Question 9
Calculate the amount of work done in rotating a dipole, of dipole moment $3x10^{-8}$ cm. from its position of stable equilibrium to the position of unstable equilibrium, in a electric field of intensity 10^4 N/C [2]

Answer:
$P = 3 \times 10^{-8}$ cm ; $E = 10^4 \frac{N}{C}$
At stable equilibrium $(\theta_1) = 0°$
At unstable equilibrium $(\theta_2) = 180°$
Work done in rotating dipole is given by:
$W = PE(\cos\theta_1 - \cos\theta_2)$
$= (3 \times 10^{-8})(10^4)[\cos 0° - \cos 180°]$
$= 3 \times 10^{-4}[1-(-1)]$
$W = 6 \times 10^{-8}$ J

Question 10
Plot a graph showing temperature dependence of resistivity for a typical semiconductor. How is this behavior explained? [2]

Answer:
Variation of resistivity (ρ) with temperature (T) is shown below:



Explanation: In semiconductor the number density of free electrons (n) increases with increase in temperature (T) and consequently the relaxation period decreases. But the effect of increase in n has higher impact than decrease of τ. So, resistivity decreases with increase in temperature.

Question 11
When four hydrogen nuclei combine to form a helium nucleus, estimate the amount of energy in MeV released in this process of fusion. (Neglect the masses of electrons and neutrinos) [2]
Given:

i. Mass of ${}_{1}^{1}H$

ii. Mass of helium nucleus = 4.002603 u, 1u = 931 $Me\frac{V}{c^2}$

Answer:
Energy released = Δm x 931 MeV

$\Delta m = 4m\,({}_{1}^{1}H) - m({}_{2}^{4}He)$

Energy released (Q) = $[4.m\,({}_{1}^{1}H) - m({}_{2}^{4}He)]$ x 931 MeV

=[4 x 1.007825 - 4.002603] x 931 MeV = 26.72 MeV.

Question 12
For an amplitude modulated wave, the maximum amplitude is found to be 10V while the minimum amplitude is 2V. Calculate the modulation index. Why modulation index is generally kept less than one? [2]

Answer:
A_{max} = 10V
A_{min} = 2 V

Modulation index = $\frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

= $\frac{10-2}{10+2}$

$= \frac{8}{12}$

$= 0.67$

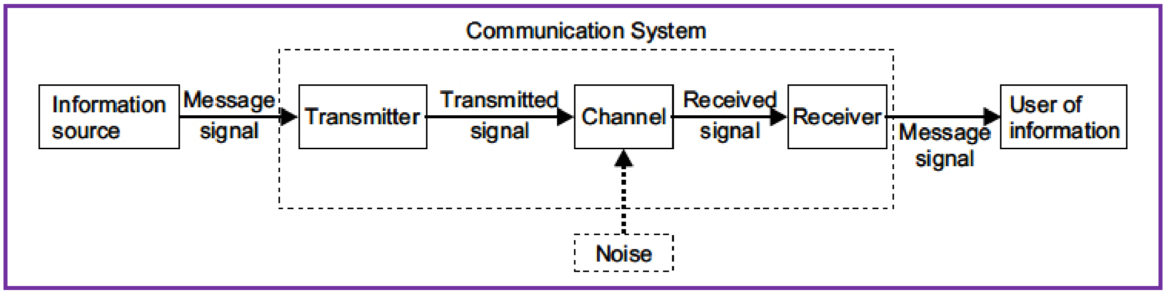
Generally, the modulation index is kept less than one to avoid distortion.

Question 13

Draw a block diagram showing the important components in a communication system. What is the function of a transducer? [2]

Answer:

Block diagram of communication system:



Function of a transducer is to convert one form of energy into another form.

Question 14

Explain the following: [2]

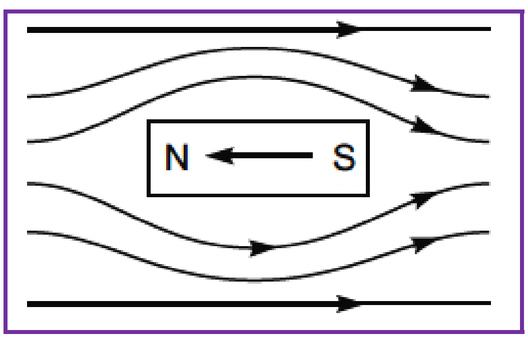
a. Why do magnetic lines of force form continuous closed loops?

b. Why are the field lines repelled (expelled) when a diamagnetic material is placed in an external uniform magnetic field?

Answer:

i. Magnetic lines of force form continuous closed loops because a magnet is always a dipole and as a result, the net magnetic flux of a magnet is always zero.

ii. When a diamagnetic substance is placed in an external magnetic field, a feeble magnetism is induced in opposite direction. So, magnetic lines of force are repelled.



OR

a. Name the three elements of the Earth's magnetic field.

b. Where on the surface of the Earth is the verticle component of the Earth's magnetic field zero?

Answer:

i. Elements of earth's magnetic field:
   a. Angle of declination (θ)
   b. Angle of dip (δ)
   c. Horizontal component of earth's magnetic field (B_H)

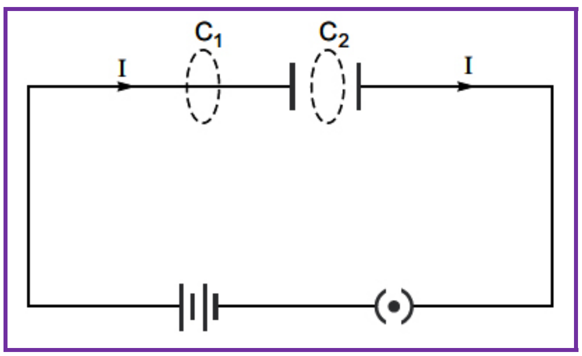
ii. At equator.

Question 15

Show how the equation for Ampere's circuital law, viz. $\oint \vec{B}.\vec{dl} = \mu_0 I$ is modified in the presence of displacement current. [2]

Answer:

Displacement current and generalised Ampere's Circuital Law: Consider a parallel plate capacitor, being charged by a battery. A time varying current is flowing through the capacitor. If we consider only the conduction current I, then we apply Ampere's Circuital Law to two closed loops C_1 and C_2, then we get



$\oint_{C_1} \vec{B}.\vec{dl} = \mu_0 I$ ....(i)

and $\oint_{C_2} \vec{B}.\vec{dl} = 0$ ....(ii)

Since there cannot be any conduction current in region between the capacitor plates. As C_1 and C_2 are very close, we must expect

$\oint_{C_1} \vec{B}.\vec{dl} = \oint_{C_2} \vec{B}.\vec{dl}$ ....(iii)

But this condition is violated by equations (i) and (ii). Hence Ampere's Circuital Law seems to be inconsistent in this case. Therefore, Maxwell postulated the existence of displacement current which is produced by time varying electric field. If σ (t) is the surface charge density on capacitor plates and q(t) is the charge, then time varying electric field E (t)=

$\frac{\sigma(t)}{\varepsilon_0} = \frac{q(t)}{A\varepsilon_0}$, where A is area of each plate.

$\frac{dE}{dt} = \frac{1}{A\varepsilon_0}\frac{dg(t)}{dt}$

or, $\frac{dg(t)}{dt} = \varepsilon_0 A \frac{dE}{dt}$

This is expression for displacement current (I_d).
Applying Kirchhoff's first law at power P, we get $I = I_d$
Hence, equation (i) and (ii) take the forms

$\oint_{C_1} \vec{B}.\vec{dl} = \mu_0 I$ and $\oint_{C_1} \vec{B}.\vec{dl} = \mu_0 I_d = \mu_0 I$

The total current is the sum of the conduction current and displacement current. Thus, modified form of Ampere's circuital law is

$$\oint \vec{B}.\vec{dl} = \mu_0 (I + I_d) = \mu_0 (I + \varepsilon_0 A \frac{dE}{dt})$$

But EA = Electric flux ϕ_E

$$\therefore \oint \vec{B}.\vec{dl} = \mu_0 (I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt})$$

Question 16

Current in a circuit falls steadily from 5.0 A to 0.0 A in 100 ms. If an average e.m.f of 200V is induced, calculated the self-inductance of the circuit. [2]

Answer:

Change in current (ΔI) =(0.0 -5.0) A = -5.0A
Time taken (Δt) =100 x10^{-3} S
Induced emf (e) = 200 V
Induced emf (e) is given by

$$e = \frac{\Delta\phi}{\Delta t}$$

$$= -\frac{\Delta(LI)}{\Delta t} \qquad (\phi = LI)$$

$$e = -L\frac{\Delta I}{\Delta t}$$

$$\text{or } L = -e.\frac{\Delta I}{\Delta t}$$

$$= -\frac{(200).(100 \times 10^{-3})}{(-5.0)}$$

L = 4.0 H

Question 17

[2]

a. You are required to select a carbon resistor of resistance 47 kΩ $\pm$10% from a large collection. What should be the sequence of color bands used to code it?

Answer:

Resistance = 47 kΩ ±10% = 47 x10^3 Ω ±10%
Sequence of color should be:
Yellow, Violet, Orange and Silver

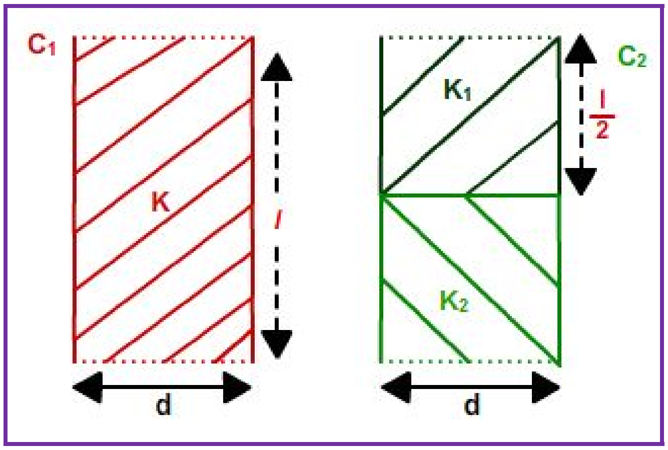
b. Write two characteristic of manganin which make it suitable for making standard resistance.

Answer:

(i) Very low temperature coefficient of resistance.
(ii) High resistivity

Question 18

Two identical parallel plate (air) capacitors C_1 and C_2 have capacitances C each. The space between their plates in now filled with dielectrics as shown. If the two capacitors still have equal capacitance, obtain the relation between dielectric constants K. K_1 and K_2. [2]



Answer:

Let $A \rightarrow$ area of each plate.

Let initially $C_1 = C = \frac{\epsilon_0 A}{d} = C_2$

After inserting respective dielectric slabs:

$C'_1 = KC$

And $C'_2 = K_1 \frac{\epsilon_0 (\frac{A}{2})}{d} + \frac{K_2 \epsilon_0 (\frac{A}{2})}{d}$ ....(i)

$= \frac{\epsilon_0 A}{2d}(K_1 + K_2)$

$C'_2 = \frac{C}{2}(K_1 + K_2)$ ....(ii)

From (i) and (ii)

$C'_1 = C'_2$

$KC = \frac{C}{2}(K_1 + K_2)$

$K = \frac{1}{2}(K_1 + K_2)$

Question 19

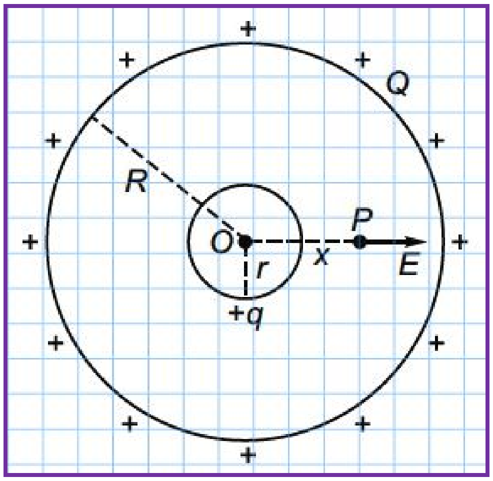
State the principle of the device that can build up high voltages of the order of a few million volts. Draw its labelled diagram. A stage reaches in this device when the potential at the outer sphere cannot be increased further by piling up more charge on it. [3]

Answer:

This device is Van de Graaff generator. **Principle:** Suppose we have a large spherical conducting shell of radius R, carrying charge Q.

The charge spreads uniformly over whole surface of the shell. Now suppose a small conducting sphere of radius ‘r’ is introduced inside the spherical shell and placed at its center, so that both the sphere and shell have same center O.

The electric field in the region inside the small sphere and large shell is due to charge +q only, so electric field strength at a distance x from the center O is



$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$, directed radially outward

The potential difference between the sphere and the shell

$V(r) - U(R) = -\int_R^r \vec{E}.d\vec{x}$

$= -\int_R^r \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}dx$

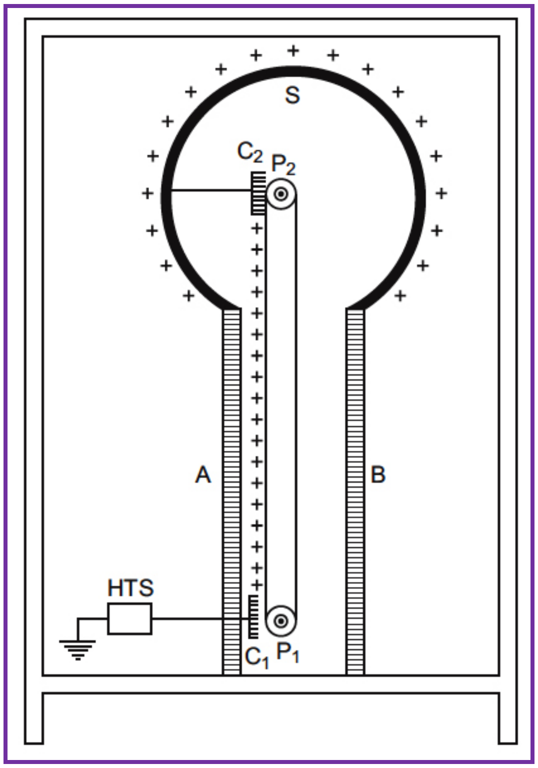
$= -\frac{1}{4\pi\varepsilon_0}\left[\frac{x^{-1}}{-1}\right]_R^r = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$

This is independent of charge Q on the large spherical shell. As r < R; V(r) -U (R) is positive. As charge flows from higher to lower potentials therefore, if we connect the small sphere and large shell by a conducting wire, the charge flows from sphere to outer shell whatsoever the charge on outer shell may be.

This forms the principle of Van de Graaff generator. The maximum charge that may be given to outer shell which may cause discharge in air.

Working: When comb C_1 is given very high potential, then it produces ions in its vicinity, due to action of sharp points. The positive ions, so produced, get sprayed on the belt due to the repulsion between positive ions and comb C_1. These positive ions are carried upward by the moving belt. The pointed end of C_2 just touches the belt.

The comb C_2 collects positive charge from the belt which immediately moves to the outer surface of sphere S. As the belt goes on revolving, it continues to take (+) charge upward, which is collected by comb C_2 and transferred to outer surface of sphere S. Thus the outer surface of metallic sphere S gains positive charge continuously and its potential rises to a very high value.



When the potential of a metallic sphere gains very high value, the dielectric strength of surrounding air breaks down and its charge begins to leak, to the surrounding air. The maximum potential is reached when the rate of leakage of charge becomes equal to the rate of charge transferred to the sphere.

To prevent leakage of charge from the sphere, the generator is completely enclosed in an earthed connected steel tank which is filled with air under high pressure. Van de Graaff generator is used to accelerate stream of charged particles to very high velocities. Such a generator is installed at IIT Kanpur which accelerates charged particles upto 2 MeV energy.

Question 20

Light wavelength 2000 $\overset{o}{A}$ falls on a metal surface of work function 4.2 eV. What is the kinetic energy (in eV) of the fastest electrons emitted from the surface?

Answer:

$\lambda = 2000\,\overset{o}{A} = 2000 \times 10^{-10}\,m$

$W_0 = 4.2\,eV$

$h = 6.63 \times 10^{-34}$

$\frac{hc}{\lambda} = W_0 + KE$

or, K.E. = $\frac{hc}{\lambda} - W_0$

$= \frac{(6.63 \times 10^{-34})x(3x10^{8})}{(2000 \times 10^{-10})} x \frac{1}{1.6 \times 10^{-19}} eV - 4.2\,eV$

= (6.2 – 4.2) eV

= 2.0 eV

a. What will be the change in the energy of the emitted electrons if the intensity of light with same wavelength is doubled?

Answer:
The energy of the emitted electrons does not depend upon intensity of incident light, hence the energy remains unchanged.

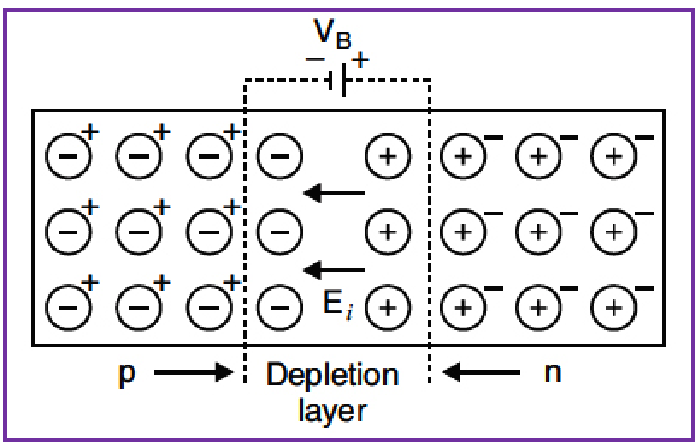
b. If the same light falls on another surface of work function 6.5 eV, what will be the energy of emitted electrons? [3]

Answer:
For this surface, electrons will not be emitted as the energy of incident light (6.2 eV) is less than the work function (6.5 eV) of the surface.

Question 21
Name the important processes that occur during the formation of a p-n junction. Explain briefly, with the help of a suitable diagram, how a p-n junction is formed. Define the term 'barrier potential' [3]

Answer:
At the junction there is diffusion of charge carriers due to thermal agitation; so that some of electrons of n-region diffuse to p-region while some of holes of p-region diffuse into n-region. Some charge carriers combine with opposite charges to neutralise each other.

Thus near the junction there is an excess of positively charged ions in n-region and an excess of negatively charged ions in p-region. This sets up a potential difference called potential barrier and hence an internal electric field E_i across the junctions.

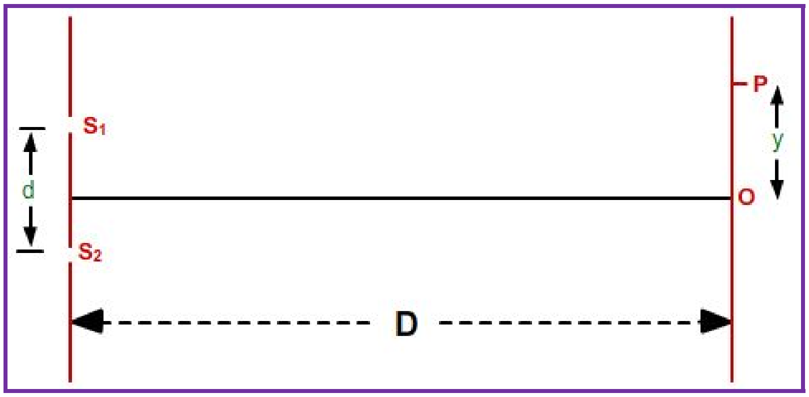


Barrier potential: During the formation of a p-n junction the electrons diffuse from n region to p-region and holes diffuse from p-region to n-region. This forms recombination of charge carriers. In this process immobile positive ions are collected at a junction toward n region and negative ions at a junction toward p-region.

This causes a p.d. across the unbiased junction. This is called potential barrier or barrier potential.

Question 22

The intensity at the central maxima (O) in a Young's double slit experiment is I_0. If the distance OP equals one-third of the fringe width of the pattern, show that the intensity at point P would be $\frac{I_0}{4}$ [3]



Answer:

Fringe width (β) = $\frac{\lambda D}{d}$

$y = \frac{\beta}{3} = \frac{\lambda D}{3d}$

Path diff (Δp) = $\frac{yd}{D} \Rightarrow \Delta p$

$= \frac{\lambda D}{3d} . \frac{d}{D}$

$= \frac{\lambda}{3}$

$\Delta\phi = \frac{2\pi}{\lambda} . \Delta p$

$= \frac{2\pi}{\lambda} . \frac{\pi}{3}$

$= \frac{2\pi}{3}$

Intensity at point P = I_0

$\cos^2 \Delta\phi = I_0 \left[\cos\frac{2\pi}{3}\right]^2$

$= I_0 \left(\frac{1}{2}\right)^2$

$= \frac{I_0}{4}$

OR

In the experiment on diffraction due to a single slit, show that [3]

a. The intensity of diffraction fringes decreases as the order (n) increases.

Answer:

In diffraction due to a single slit the path difference is given by:

$\Delta x = \alpha \sin\theta$ where, α is the width of the slit

For maxima: $\Delta x = (2x + 1)\,\dfrac{\lambda}{2}$

$\Delta x = a \sin\theta = (2x + 1)\,\dfrac{\lambda}{2}$

For $n = 2$, $\Delta x = \dfrac{3.\lambda}{2}$

Let us divide the slit into three equal parts. If we take first two parts of slit, the path difference between rays diffracted from the extreme ends of first two parts

$$\frac{2}{3}\alpha \sin\theta$$

$$= \frac{2}{3}\alpha \, x \, \frac{3\lambda}{2\alpha}$$

$$= \lambda$$

Then the first two parts will have a path difference of $\dfrac{\lambda}{2}$ and cancel the effect of each other.

The remaining third part will contribute to the intensity at a point between two minima. This is called first secondary maxima. In similar manner we can show that the intensity of the other secondary maxima will go on decreasing.

b. Angular width of the central maximum is twice that of the first order secondary maximum.

Answer:

The general maxima has between first minima on either side of the central maxima. We know for first minima.

$\alpha \sin\theta = \lambda \Rightarrow \alpha\theta = \lambda$

$\because$ for small angle $\sin\theta \approx \theta$

$$\tan\theta = \frac{y_1}{D}$$

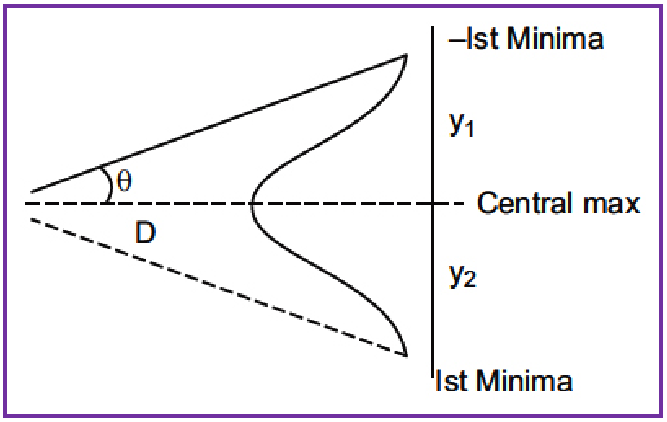
$$\Rightarrow \theta = \frac{y_1}{D}$$

$$\Rightarrow \frac{\lambda}{\alpha} D = y_1 = y_2$$

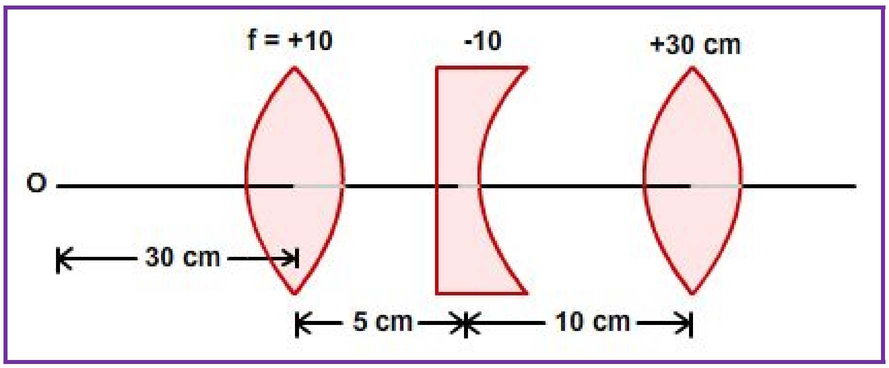
Hence, whole width on secondary maxima on one side is $\dfrac{\lambda D}{d}$.

The angular width of the central maxima = $\dfrac{2\lambda D}{\alpha}$

So, angular width of the central maxima is twice that of the first order secondary maximum.



Question 23
Find the position of the image formed of the object ‘O’ by the lens combination given in the figure.



[3]

Answer:
For first lens, u1 = -30 cm, f_1 = +10 cm

$\therefore$ From lens formula $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1}$$

$$= \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} \Rightarrow v_1$$

= 15 cm

This means that the image formed by first lens is at a distance of 15 cm to the right of first lens. This image serves as a virtual object for second lens.
For second lens, f_2 = -10 cm, u_2 =15 -5 = +10 cm

$$\therefore \frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$$

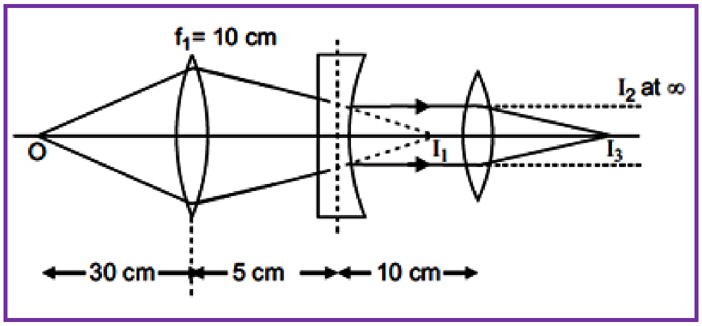
$$= \frac{1}{10} + \frac{1}{10} \Rightarrow v_2 \infty$$

This means that the real image is formed by second lens at infinite distance. This acts as an object for third lens.
For third lens, f_3 = + 30 cm, $u_3 = \infty$

From lens formula $\frac{1}{v_2} = \frac{1}{f_3} + \frac{1}{u_3} = \frac{1}{30} + \frac{1}{\infty}$

$\Rightarrow v_3 = 30cm$

i.e., final image is formed at a distance 30 cm to the right of third lens. The ray diagram of formation of image is shown in figure.

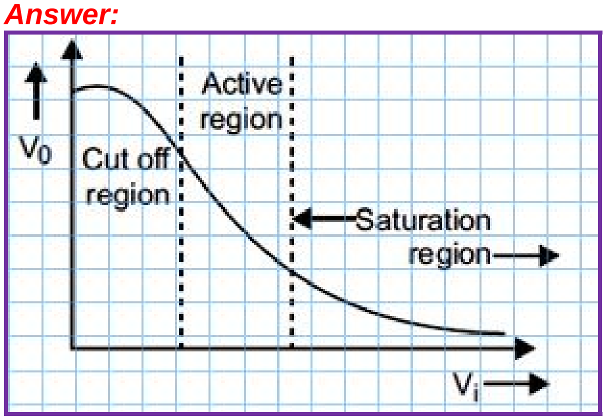


Question 24

Draw transfer characteristics of a common emitter n-p-n transistor. Point out the region in which the transistor operates as an amplifier. Define the following terms used in transistor amplifiers.[3]

i. Input resistance
ii. Output resistance
iii. Current amplifier factor

Answer:



In active region the transistor is used as an amplifier.

1. ***Input Resistance:*** It is the ratio of change in emitter base voltage (ΔV_{EB})to the corresponding change in emitter current (ΔI_E)at constant collector-base voltage (V_{CB}) i. e.

   Input resistance $r_i = \left(\frac{\Delta V_{EB}}{\Delta I_E}\right)_{V_{CB}=\text{constant}}$

   Physically input resistance is the hindrance offered to the signal current. The input resistance is very small, of the order of a few ohms, because a small change in V_{EB} causes a large change in I_E.

2. ***Output Resistance:*** It is the ratio of change in collector-base voltage to the corresponding change in collector current at constant emitter current I_E.

   i.e., $r_o = \left(\frac{\Delta V_{CB}}{\Delta I_C}\right)_{V_{EB}=\text{constant}}$

   The output resistance is very high, of the order of several-tens kilo ohm because a large change in collector-base voltage causes a very small change in collector current.

3. ***Current amplification factors of a transistor (α and β):***
The current gain α is defined as the ratio of change in collector current to the change in emitter current for constant value of collector voltage in common base configuration, i.e.,

$$\alpha = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_C = \text{constant}} \quad \text{..................(i)}$$

Practical value of α ranges from 0.9 to 0.99 for junction transistor. The current gain β is defined as the ratio of change in collector current to the change in base current for constant value of collector voltage in common emitter configuration i. e.,

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_C = \text{constant}} \quad \text{..................(ii)}$$

The value of β ranges from 20 to 200.

The current gains α and β are related as $\alpha = \dfrac{\beta}{1+\beta}$

or. $\beta = \dfrac{\alpha}{1-\alpha}$ ....(iii)

Question 25

a. Light passes through two polaroids P_1 and P_2 with its pass axis of P_2 making an angle θ with the pass axis of P_1. For what value of θ is the intensity of emergent light zero?

Answer:

At θ = 90°, the intensity of emergent light is zero.

b. A third polaroid is placed between P_1 and P_2 with its pass axis making an angle β with the pass axis of P_1. Find a value of β for which the intensity of light emerging from P_2 is $\dfrac{I_0}{8}$, where I_0 is the intensity of light on the polaroid P_1. [3]

Answer:

Intensity of light coming out polariser $P_1 = \dfrac{I_0}{2}$

Intensity of light coming out from $P_2 = \dfrac{I_0}{2}\cos^2\beta\cos^2(90-\beta)$

$$= \frac{I_0}{2}\cos^2\beta.\sin^2\beta$$

$$= \frac{I_0}{2}\left[\frac{(2\cos^2\beta.\sin^2\beta)^2}{2}\right]$$

$$I = \frac{I_0}{8}(\sin 2\beta)^2$$

But it is given that intensity transmitted from P_2 is

$$I = \frac{I_0}{8}$$

So, $\dfrac{I_0}{8} = \dfrac{I_0}{8}(\sin 2\beta)^2$

$$or, (\sin 2\beta)^2 = 1$$

$$\sin 2\beta = \sin\frac{\pi}{2} \Rightarrow \beta = \frac{\pi}{4}$$

Question 26

Using the postulates of Bohr's model of hydrogen atom, obtain an expression from the higher energy state with quantum number n_i to the lower energy state with quantum number $n_f (n_f < n_i)$

[3]

Answer:

Suppose m be the mass of an electron and v be its speed in nth orbit of radius r. The centripetal force for revolution is produced by electrostatic attraction between electron and nucleus.

$$\frac{m\upsilon^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \text{ .... (i)}$$

$$\text{or, } m\upsilon^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

so, Kinetic energy [K]= $\frac{1}{2}m\upsilon^2$

$$K = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

Potential energy = $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r}$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

Total energy, E = KE + PE

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}\right)$$

$$E = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

For nth orbit, E can be written as E_n

$$\text{So, } E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n}$$

Again from Bohr's postulate for quantization of angular momentum

$$m\upsilon r = \frac{nh}{2\pi}$$

$$\upsilon = \frac{nh}{2\pi mr}$$

Substituting this value of υ in equation (i), we get

$$\frac{m}{r}\left[\frac{nh}{2\pi mr}\right]^2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

$$\text{or, } r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

$$\text{or, } r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

Substituting value of r_n in equation (ii), we get

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2\left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right)}$$

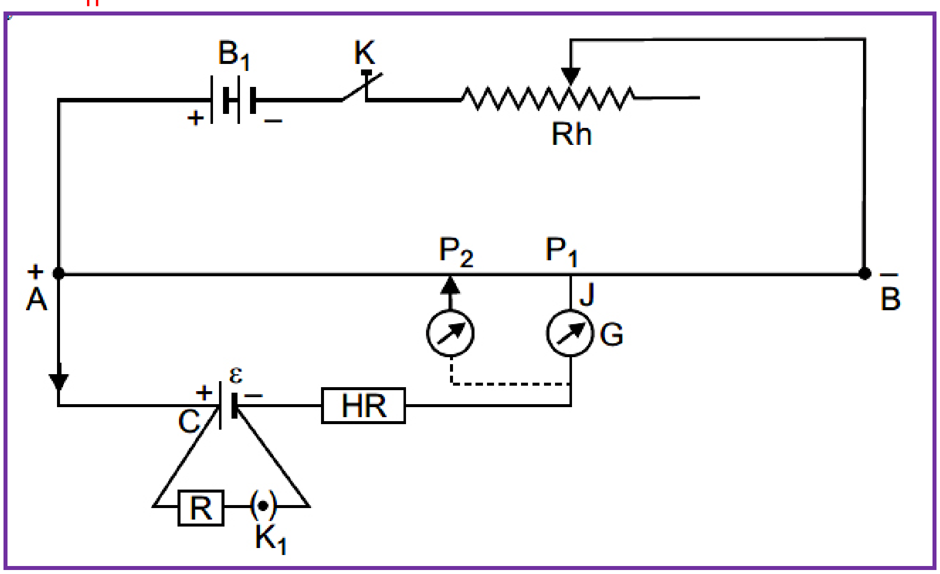
$$= -\frac{mZ^2e^2}{8\varepsilon_0 h^2 n^2}$$

$$\text{or, } E_n = -\frac{Z^2Rhc}{n^2}$$

where $R = \dfrac{me^4}{8\varepsilon_0^2 ch^3}$

R is called Rydberg constant.
For hydrogen atom Z =1,

$$E_n = \frac{-Rch}{n^2}$$



If n_i and n_f are the quantum numbers of initial and final states and E_i & E_f are energies of electron in H-atom in initial and final state, we have

$$E_i = \frac{-Rhc}{n_i^2} \text{ and } E_f = \frac{-Rhc}{n_f^2}$$

If ν is the frequency of emitted radiation. We get

$$\nu = \frac{E_i - E_f}{h}$$

$$\nu = \frac{-Rc}{n_i^2} - \left(\frac{-Rc}{n_f^2}\right)$$

$$\nu = Rc\left[\frac{1}{n_f^2} - \frac{-Rc}{n_i^2}\right]$$

Question 27

State the underlying principle of a potentiometer. Describe briefly, given the necessary circuit diagram, how a potentiometer is used to measure the internal resistance of a given cell. [3]

Answer:

Principle of potentiometer: If constant current is flowing through a wire of uniform area of cross-section at constant temperature, the potential drop across- any portion of wire is directly proportional to the length of that portion

$V \propto l$

Determination of internal resistance of potentiometer.

i. Initially key K is closed and a potential difference is applied across the wire AB. Now rheostat (Rh) is so adjusted that on touching the jockey J at ends A and B of potentiometer wire, the deflection in the galvanometer is on both sides. Suppose that in this position the potential gradient on the wire is k.

ii. Now key K_1 is kept open and the position of null deflection is obtained by sliding and pressing the jockey on the wire. Let this position be P_1 and $AP_1 = l_1$.In this situation the cell is in open circuit, therefore the terminal potential difference will be equal to the emf of cell, i.e.,
emf $\varepsilon = kl_1$ ...(i)

iii. Now a suitable resistance R is taken in the resistance box and key K_1 is closed. Again, the position of null point is obtained on the wire by using jockey J. Let this position on wire be P_2 and $AP_2 = l_2$.

In this situation the cell is in closed circuit, therefore the terminal potential difference (V) of cell will be equal to the potential difference across external resistance R, i.e.,
$V = kl_2$ ....(ii)

Dividing (i) by (ii), we get $\frac{\varepsilon}{V} = \frac{l_1}{l_2}$

$\therefore$ Internal resistance of cell, r = $\left(\frac{\varepsilon}{V} - 1\right)R = \left(\frac{l_1}{l_2} - 1\right)R$

From this formula r may be calculated.

Question 28 [5]

a. Show that a planar loop carrying a current I, having n closely wound turns and area of cross-section A, possesses a magnetic moment $\vec{m} = NI\vec{A}$.

Answer:

Torque (τ) on the loop is given by:

$\vec{\tau} = NI\vec{A}x\vec{B}$ which can be written as,

$\vec{\tau} = \vec{M}$ x $\vec{B}$

where, $\vec{M}$ is the magnetic dipole moment given by $\vec{M} = N\,I\,\vec{A}$

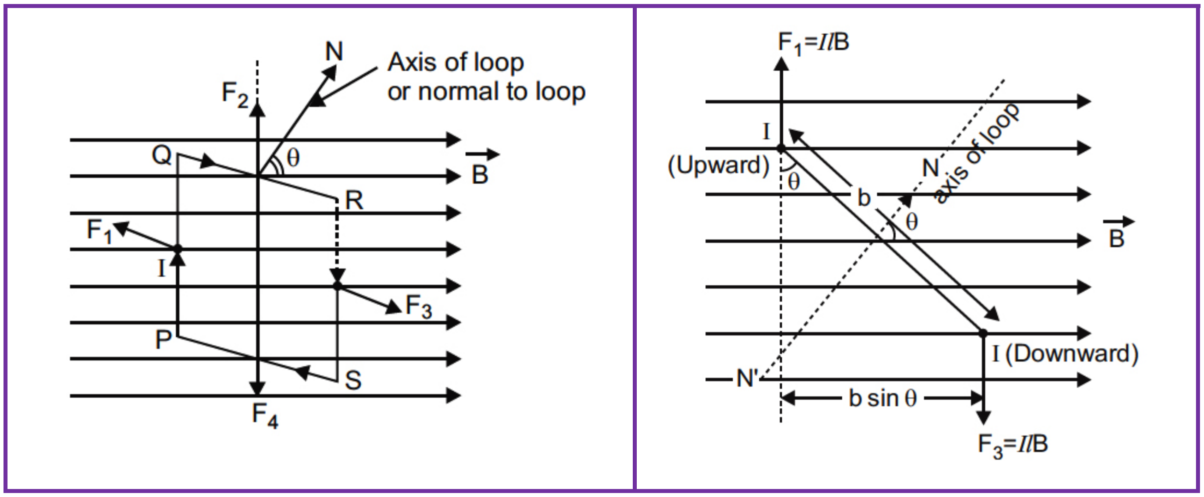
b. When this loop is placed in a magnetic field $\vec{B}$, find out the expression for the torque acting on it.

Answer:

Torque on a current carrying loop: Consider a rectangular loop PQRS of length l, breadth b suspended in a uniform magnetic field $\vec{B}$. The length of loop = PQ = RS =l and breadth = QR =SP = b.

Let at any instant the normal to the plane of loop make an angle θ with the direction of magnetic field $\vec{B}$ and I be the current in the loop. We know that a force acts on a current carrying wire placed in a magnetic field.

Therefore, each side of the loop will experience a force. The net force and torque acting on the loop will be determined by the forces acting on all sides of the loop. Suppose that the forces on sides PQ, QR, RS and SP are $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$ respectively.



The sides QR and SP make angle (90° - θ) with the direction of magnetic field. Therefore each of the forces $\vec{F_2}$ and $\vec{F_4}$ acting on these sides has same magnitude F' = Blb sin (90° - θ) = Blb cos θ. According to Fleming's left hand rule the forces $\vec{F_2}$ and $\vec{F_4}$ are equal and opposite but their line of action is same. Therefore these forces cancel each other i.e. the resultant of $\vec{F_2}$ and $\vec{F_4}$ is zero.

The sides PQ and RS of current loop are perpendicular to the magnetic field, therefore the magnitude of each of forces $\vec{F_1}$ and $\vec{F_3}$ is F = IlB sin 90° = IlB.

According to Fleming's left hand rule the forces $\vec{F_1}$ and $\vec{F_3}$ acting on sides PQ and RS are equal and opposite, but their lines of action are different; therefore the resultant force of $\vec{F_1}$ and $\vec{F_3}$ is zero, but they form a couple called the deflecting couple. When the normal to plane of loop makes an angle θ with the direction of magnetic field B, the perpendicular distance between F_1 and F_3 is b sinθ.

$\therefore$ Moment of couple or Torque,
τ = (Magnitude of one force F) x perpendicular distance =(BIl) . (b sin θ) = I (lb) B sin θ
But lb =area of loop = A (say)

$\therefore$ Torque, τ = IAB sin θ
If the loop contains N-turns, then τ = NI AB sin θ
In vector form $\vec{\tau} = NI\ \vec{A} \times \vec{B}$

Direction of torque is perpendicular to direction of area of loop as well as the direction of magnetic field i.e., along $I\ \vec{A} \times \vec{B}$.

c. A galvanometer coil of 50Ω resistance shows full scale deflection for a current of 5mA. How will you convert this galvanometer into a voltmeter of range 0 to 15V?

Answer:
G =50Ω
I_g =5 m A =5 x 10^{-3} A
V =15 V

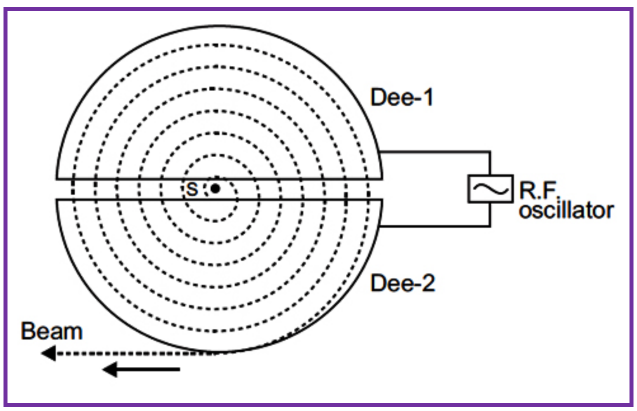
The galvanometer can be converted into a voltmeter when a high resistance R is connected in series with it. Value of R is given by: $R = \frac{V}{I_g} - G = \frac{15}{5 \times 10^{-3}} - 50$

=3000 – 50
= 29 50 Ω = 2.95 k Ω

OR

a. Draw a schematic sketch of a cyclotron, explain its working principle and deduce the expression for the kinetic energy of the ions accelerated.

Answer:
Principle: The positive ions produced from a source are accelerated. Due to the presence of perpendicular magnetic field the ion will move in a circular path. The phenomenon is continued till the ion reaches at the periphery where an auxiliary negative electrode (deflecting plate) deflects the accelerated ion on the target to be bombarded.



Expression for K.E. attained:
If R be the radius of the path and υ_{max} the velocity of the ion when it leaves the periphery, then

$$\upsilon_{max} = \frac{qBR}{m}$$

The kinetic energy of the ion when it leaves the apparatus is,

$$K.E. = \frac{1}{2} m\upsilon_{max}^2 = \frac{q^2B^2R^2}{2m}$$

When charged particle crosses the gap between dees it gains KE = q V
In one revolution, it crosses the gap twice, therefore if it completes n-revolutions before emerging the does, the kinetic energy gained=2nqV.

Thus K.E. $= \dfrac{q^2B^2R^2}{2m} = 2nqV$

b. Two long and parallel straight wires carrying currents of 2A and 5A in the opposite directions are separated by a distance of 1cm. find the nature and magnitude of the magnetic force between them.

[5]

Answer:

$I_1 = 2A$, $I_2 = 5A$, $\alpha = 1$ cm $= 1\times10^{-2}$ m

Force between two parallel wires per unit length is given by $F = \dfrac{\mu_0}{2\pi}.\dfrac{I_1 I_2}{\alpha}$

$= 2\times10^{-7} \times \dfrac{2\times5}{1\times10^{-2}} = 20\times10^{-5}$ N (Repulsive)

Question 29

[5]

a. Derive the expression for the mutual inductance of two long coaxial solenoids of same length l having radii r_1 and r_2 $(r_2 > r_1$ and $l >> r_2)$.

Answer:

Suppose there are two coils C_1 and C_2. The current I_1 is flowing in primary coil C_1; due to which an effective magnetic flux Φ_2 is linked with secondary coil C_2. By experiments

$\Phi_2 \propto I_1$ or $\Phi_2 = MI_1$ ....(i)

Where M is a constant, and is called the coefficient of mutual induction or mutual inductance.

From (i) $M = \dfrac{\Phi_2}{I_1}$ If $I_1 = 1$ ampere, M= Φ_2

i.e., the mutual inductance between two coils is numerically equal to the effective flux linkage with secondary coil, when current flowing in primary coil is 1 ampere. Mutual Inductance of Two Co-axial Solenoids: Consider two long co-axial solenoid each of length l with number of turns N_1 and N_2 wound one over the other. Number of turns per unit length in order (primary) solenoid,

n= $\dfrac{N_1}{l}$ If I_1 is the current flowing in primary solenoid, the magnetic field produced within this solenoid.

$B1 = \dfrac{\mu_0 N_1 I_1}{l}$ .... (ii)

The flux linked with each turn of inner solenoid coil is $\Phi_2 = B_1A_2$, where A_2 is the cross-sectional area of inner solenoid. The total flux linkage with inner coil of N_2 -turns.

$$\Phi_2 = N_2\phi_2 = N_2B_1A_2 = N_2\left(\frac{\mu_0 N_1 N_2}{l}\right)A_2 = \frac{\mu_0 N_1 N_2}{l}A_2I_1$$

By definition Mutual Inductance, $M_{21} = \dfrac{\Phi_2}{I_1} = \dfrac{\mu_0 N_1 N_2 A_2}{l}$

If n_1 is number of turns per unit length of outer solenoid and r_2 is radius of inner solenoid, then

$M = \mu_0 N_1 N_2 \pi r_2^2$

b. Show that mutual induction of solenoid 1 due to solenoid 2, M_{12}, is the same as that of 2 due to 1 i.e. M_{21}.

Answer:
Due to current I_1 through solenoid of radius r_1, flux linked with second solenoid $N_2\phi_2 = M_{21} I_1$ .... (i)

But flux due to current I_1 in first solenoid (using Ampere's circuital law) will be = $\frac{\mu_0 N_1 I_1}{l}$.

Hence $N_2\phi_2 = N_2(\pi r_1^2)\left(\mu_0 \frac{N_1}{l} I_1\right)$ as $l >> r_2$ and $r_2 >> r_1$

Using (1) $M_{21} = \frac{\mu_0 N_1 I_1}{l}\pi r_1^2$ which is same as expression of mutual inductance derived in part (a) above $\therefore M_{21} = M_{12}$

c. A power transmission line feeds power at 2200 V with a current of 5A to a step down transformer with its primary winding having 4000 turns. Calculate the number of turns and the current in the secondary in order to get output power at 220V.

Answer:
$V_p = 2200V.\ I_0 = 5A.\ N_p = 4000$

$V_s = 220V.\ N_s = ?\ I_s = ?$

$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$

$\frac{220}{2200} = \frac{5}{I_s} = \frac{N_s}{4000}$

$\frac{22p}{220p} = \frac{5}{I_s}$

$\frac{1}{p} = \frac{5}{I_s}$

$I_s = 50\,A$

$\frac{5}{I_s} = \frac{N_s}{4000}$

$\frac{5}{50} = \frac{N_s}{4000}$

$N_s = 400$

OR

a. An alternating voltage $v = v_m \sin \omega t$ applied to a series LCR circuit drives a current given by $i = i_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.

Answer:
$V = V_m \sin \omega t$

$i = i_m(\omega t + \phi)$

and instantaneous power, $P = Vi$

$= V_m \sin \omega t \,.\, i_0 \sin(\omega t + \phi) = V_m i_m \sin \omega t \sin(\omega t + \phi)$

$= \frac{1}{2} V_m i_m 2 \sin \omega t \sin(\omega t + \phi)$

From trigonometric formula 2 sin A sin B = cos (A -B) - cos (A + B)

$\therefore$ Instantaneous power, $P = \frac{1}{2} V_m i_m [\cos(\omega t + \phi - \omega t) - \cos(\omega t + \phi + \omega t)]$

$= \frac{1}{2} V_m i_m [\cos\phi - \cos(2\omega t + \phi)]$..........(i)

Average power for complete cycle

$\vec{P} = \frac{1}{2} V_m i_m [\cos\phi - \overline{\cos(2\omega t + \phi)}]$

where cos $\overline{(2\omega t + \phi)}$ is the mean value of cos $(2\omega t + \phi)$ over complete cycle. But for a complete cycle, $\overline{\cos(2\omega t + \phi)} = 0$

$\therefore$ Average power, $\vec{P} = \frac{1}{2} V_m i_m \cos\phi$

$= \frac{V_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos\phi$

$\vec{P} = V_{rms} i_{rms} \cos\phi$

b. For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

Answer:

The power is $P = V_{rms} i_{rms} \cos\phi$. If $\cos\phi$ is small, then current considerably increases when voltage is constant. Power loss, we know is I^2R. Hence, power loss increases.

c. Determine the current and quality factor at resonance for a series LCR circuit with L =1.00 mH, C = 1.00 nF and R = 100 Ω connected to an a.c source having peak voltage of 100V.

Answer:

I_v =? , Q =?

L =1.00 mH=1×10^{-3} H, C =1.00 nF =1×10^{-9} F

R =100Ω, E_0 =100 V

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{E_0}{Z} \begin{Bmatrix} \text{at resonance } \omega L = \frac{1}{\omega C} \\ \text{Hence } Z = R \end{Bmatrix}$$

$$\therefore I = \frac{V}{R} = \frac{100}{100}$$

I_0 =1A

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.44}{2} = 0.707A$$

$I_v = 0.707A$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{1.0\times10^{-3}}{1.0\times10^{-9}}} = \frac{1}{100} \times 10^3$$

=10

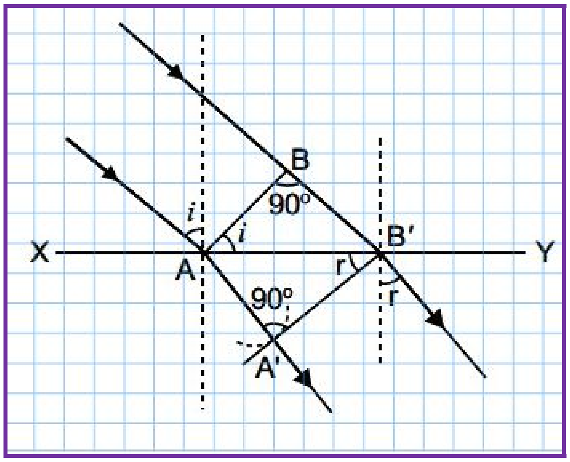
Q=10

Question 30 [5]

a. A plane wave front approaches a plane surface separating two media. If medium 'one' is potically denser and medium 'two' is optically rarer, using Huygen's principle, explain and show how a refracted wavefrond is constructed.

Answer:

When a wave starting from one homogeneous medium enters the homogeneous medium, it is deviated from its path. This phenomenon is called refraction. In trans versing from first medium to another medium, the frequency of wave remains unchanged but its speed and the wavelength both are changed. Let XY be a surface separating the two media '1' and '2'. Let υ_1 and υ_2 be the speeds of waves in these media.



Suppose a plane wavefront AB in first medium is incident obliquely on the boundary surface XY and its end A touches the surface at A at time t = 0 while the other end B reaches the surface at point B' after time-interval t. Clearly BB' = υ_1t. As the wavefront AB advances, it strikes the points between A and B' of boundary surface. According to Huygen's principle, secondary spherical wavelets originate from these points, which travel with speed υ_1 in the first medium and speed υ_2 in the second medium.

First of all secondary wavelet starts from A, which traverses a distance AA' (= υ_2 t) in second medium in time t. In the same time-interval t, the point of wave front traverses a distance BB' (= υ_1 t) in first medium and reaches B', from, where the secondary wavelet now starts. Clearly BB' = υ_1 t and AA¢ = υ_2 t.

Assuming A as centre, we draw a spherical arc of radius AA' (= υ_2 t) and draw tangent B¢ A¢ on this arc from B'. As the incident wavefront AB advances, the secondary wavelets start from points between A and B', one after the other and will touch A' B' simultaneously. According to Huygen's principle A' B' is the new position of wavefront AB in the second medium. Hence A' B' will be the refracted wavefront.

b. Hence verify Snell's law.

Answer:

Proof of Snell's law of Refraction using Huygen's wave theory: When a wave starting from one homogeneous

First law: As AB, A' B' and surface XY are in the plane of paper, therefore the perpendicular drawn on them will be in the same plane. As the lines drawn normal to wave front denote the rays, therefore we may say that the incident ray, refracted ray and the normal at the point of incidence all lie in the same plane.

This is the first law of refraction.

Second law: Let the incident wavefront AB and refracted wavefront A' B' make angles i and r respectively with refracting surface XY.

In right-angled triangle AB' B, $\angle$ABB' = 90°

$$\therefore \sin i = \sin\angle BAB' = \frac{BB'}{AB'} = \frac{\upsilon_1 t}{AB'} \text{ .... (i)}$$

Similarly in right-angled triangle AA' B', $\angle$ AA' B' = 90°

$$\therefore \sin r = \sin\angle AB'A' = \frac{AA'}{AB'} = \frac{\upsilon_2 t}{AB'} \text{ .... (ii)}$$

Dividing equation (i) by (ii), we get

$$\frac{\sin i}{\sin r} = \frac{\upsilon_1}{\upsilon_2} = \text{constant .... (iii)}$$

As the rays are always normal to the wavefront, therefore the incident and refracted rays make angles i and r with the normal drawn on the surface XY i.e. i and r are the angle of incidence and angle of refraction respectively. According to equation (3):

The ratio of sine of angle of incidence and the sine of angle of refraction is a constant and is equal to the ratio of velocities of waves in the two media. This is the second law of refraction, and is called the Snell's law.

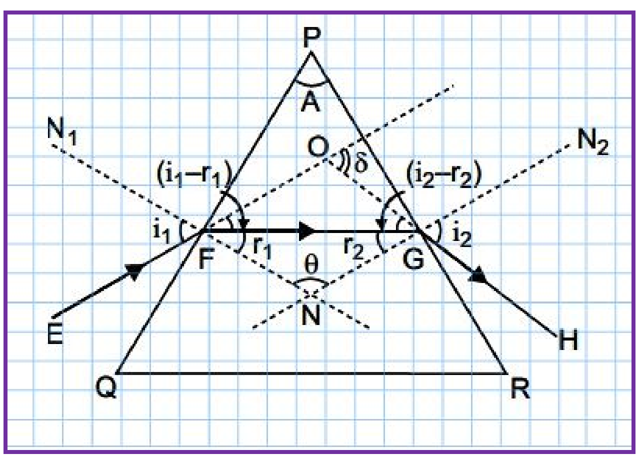
c. When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain.

Answer:
No. Because energy of wave depends on its frequency and not on its speed.

OR

a. A ray of monochromatic light is incident on one of the faces of an equilateral triangular prism of refracting angle A. trace the path of ray passing through the prism. Hence derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and its refracting angle.

Answer:
Let PQR be the principal section of the prism. The refracting angle of the prism is A. A ray of monochromatic light EF is incident on face PQ at angle of incidence i_1. The refractive index of material of prism for this ray is n.



This ray enters from rarer to denser medium and so is deviated towards the normal FN and gets refracted along the direction FG. The angle of refraction for this face is r_1. The refracted ray FG becomes incident on face PR and is refracted away from the normal GN_2 and emerges in the direction GH.

The angle of incidence on this face is r_2 (into prism) and angle of refraction (into air) is i_2. The incident ray EF and emergent ray GH when produced meet at O. The angle between these two rays is called angle of deviation 'δ'.

$\angle OFG = i_1 - r_1$ and $\angle OGF = i_2 - r_2$

In Δ FOG, d is exterior angle

$\therefore \delta = \angle OFG + \angle OGF = (i_1 - r_1) + (i_2 - r_2)$

$= (i_1 + r_1) - (i_2 + r_2)$ .... (i)

The normals FN_1 and GN_2 on faces PQ and PR respectively, when produced meet at N. Let $\angle FNG = \theta$.

In Δ FGN, $r_1 + r_2 + \theta = 180°$ .... (ii)

In quadrilateral PFNG, $\angle$ PFN = 90°, $\angle$ PGN = 90°

$\therefore A + 90^0 + \theta + 90^0 = 360^0$ or, $A + \theta = 180^0$ .... (iii)

Comparing (ii) and (iii), $r_1 + r_2 = A$ .... (iv)

Substituting this value in (i), we get

$\delta = i_1 + i_2 - A$ ....(v)

or, $i_1 + i_2 = A + \delta$ ....(vi)

From Snell's law $n = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2}$ .... (vii)

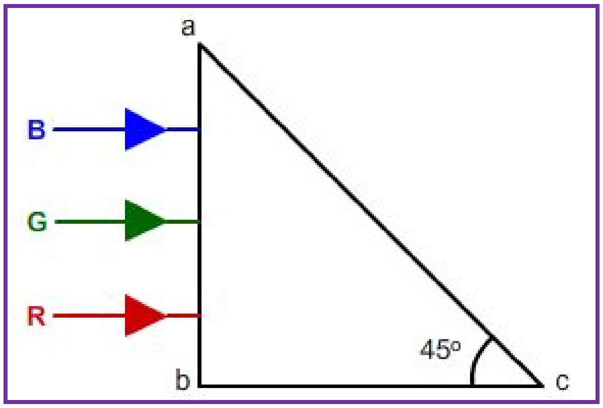
For minimum deviation i_1 and i_2 become coincident, i.e., $i_1 = i_2 = i$ (say) So from (vii) $r_1 = r_2 = r$ (say)

Hence from (iv) and (vi), we get r+r=A or $r = \frac{A}{2}$

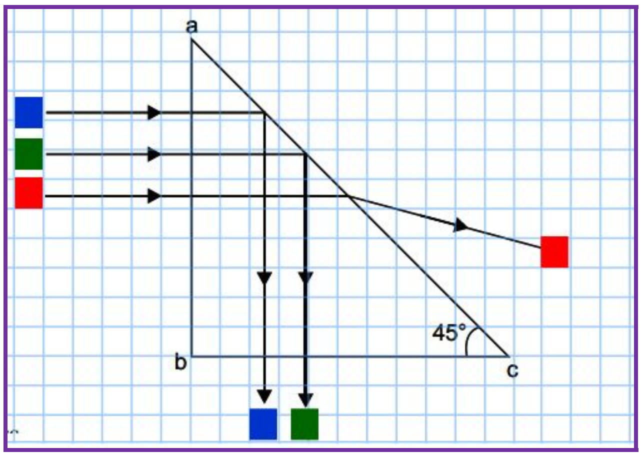
and i + i =A+ δ_m or $i = \frac{A + \delta_m}{2}$

Hence from Snell's law, $n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

b. Three light rays (R), green (G) and blue (B) are incident on the right angled prism abc at face ab. The refractive indices of the material of the prism for red, green and blue wavelengths are respectively 1.39, 1.44 and 1.47. Trace the paths of these rays reasoning out the difference in their behavior.



Answer:



Angle of incidence at face ac for all three colors,
i = 45°

Refractive index corresponding to critical angle 45° is $\mu = \dfrac{1}{\sin 45^0} = \sqrt{2} = 1.414$

The ray will be transmitted through face 'ac' if i <i_c. This condition is satisfied for red color (μ =1.39). So only red ray will be transmitted, blue and Green rays will be totally reflected.