2008

Section: A
Questions: 1 – 7 ii-xi

Section: B
Questions: 8 – 12 xii-xiv

Section: C
Questions: 13 – 16 xv-xviii

Section A *(Question numbers 1 to 7)*

Question: 1 [3 × 9 = 27]

i. Find x and y, if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x - y \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

Answer:

Given: $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ....(i)

$x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ....(ii)

Adding (i) and (ii), $2x = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\therefore\ x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Subtracting (ii) from (i), $2y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$\therefore\ y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

ii. Find the equation of the straight line through origin and passing through the intersection of lines $x - 2y = 0$ and $3x + 5y + 6 = 0$. **

i. A straight line $2x + y + p = 0$ is a focal chord of the parabola $y2 = -8x$. Find the value of p.**

ii. If $y = e^{\sin x^2}$ find $\frac{dy}{dx}$.

Answer:

$\frac{dy}{dx} = e^{\sin x^2}.\cos x^2.2x$

iii. Evaluate $\int x^2 \left(e^{x^3}\right)\cos\left(2e^{x^3}\right)dx$.

Answer:

$I = \int x^2 e^{x^3} \cos(2x^3)dx = \frac{1}{3}\int e^z \cos 2z\,dz \quad z = x^3$. Applying by parts get the result.

iv. If $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$, find $\frac{dy}{dx}$.

Answer:

Given: $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$,

$$\Rightarrow y = \sqrt{\frac{(1-\cos x)(1-\cos x)}{1-\cos^2 x}}$$

$$= \frac{1-\cos x}{\sin x}$$

$$\Rightarrow y = \operatorname{cosec} x - \cot x$$

$$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x$$

$$= -\operatorname{cosec} x\left[\cot x - \operatorname{cosec} x\right].$$

v. Evaluate $\int \frac{x^2}{\left(x^2-4\right)} dx.$

Answer:

Given: $\int \frac{x^2}{x^2-4} dx$

$$\therefore \int \frac{x^2-4+4}{x^2-4} dx$$

$$\Rightarrow \int \left(1+\frac{4}{x^2-4}\right) dx = x + 4 \times \frac{1}{2 \times 2} \log \left|\frac{x-2}{x+2}\right|$$

$$= x + \log \left|\frac{x-2}{x+2}\right| + c.$$

vi. Find the equation of tangents to the hyperbola $3x^2 - y^2 = 3$ which are perpendicular to the line $x + 3y = 2$ **

vii. In a single throw of two dice, find the probability of getting a total of at most 9.

Answer:

Total no. of ways = 36

The ways in favour are:

$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$

$(2,10(2,2),(2,3),(2,4),(2,5),(2,6)$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$

$(4,1),(4,2),(4,3),(4,4),(4,5)$

$(5,1),(5,2),(5,3),(5,4)$

$(6,1),(6,2),(6,3)$

$\therefore$ Total no. of ways in favour = 30

$\therefore$ Required probability $= \frac{30}{36} = \frac{5}{6}.$

viii. If the standard deviation of the numbers, 2, 3, 11 and x is, $3\frac{1}{2}$ find the value x. **

ix. Find the value of x and y, given that $(x+iy)(2-3i)=4+i$.

Answer:

Given : $(x+iy)(2-3i)=4+i$

$\Rightarrow (2x+3y)+i(2y-3x)=4+i$

$2x+3y=4..........(i)$

$-3x+2y=1..........(ii)$

Solving these equations we get,

$x=\dfrac{5}{13}$ and $y=\dfrac{14}{13}$.

x. Solve the differential equation : $(x+1)\dfrac{dy}{dx}-y=e^{3x}(x+1)^2$.

Answer:

Given equation is, $(x+1)\dfrac{dy}{dx}-y=e^{3x}(x+1)^2$

Dividing the whole equation by $(x+1)$

$\dfrac{dy}{dx}-\dfrac{1}{(x+1)}y=e^{3x}(x+1)$

$\therefore$ I.F. $=e^{-\int\frac{1}{x+1}dx}$

$=e^{-\log(x+1)}$

$=\dfrac{1}{x+1}$

Multiplying by I.F. to the given equation $\dfrac{1}{x+1}\left[\dfrac{dy}{dx}-\dfrac{1}{x+1}y\right]=e^{3x}.(x+1).\dfrac{1}{(x+1)}$

Integrating on both sides $y.\dfrac{1}{(x+1)}=\int e^{3x}dx$

$\Rightarrow \dfrac{y}{x+1}=\dfrac{e^{3x}}{3}+c$

$\Rightarrow y=\dfrac{e^{3x}}{3}(x+1)+c(x+1)$.

Question: 2 [5 × 2=10]

a. If $A=\begin{bmatrix}3\\5\\2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 4\end{bmatrix}$ find (AB).

Answer:

Given: $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$

$R_3 \to R_3 \to xR_1 \to yR_2$

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ 0 & 0 & -ax^2-bxy-bxy-cy^2 \end{vmatrix}$$

Now expanding w. r. t. R_3 ,

$= -\left(ax^2+2bxy+cy^2\right)\left(ac-b^2\right)$

$= \left(b^2-ac\right)\left(ax^2+2bxy+cy^2\right).$

b. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations:

$x+2y-3z=-4$

$2x+3y+2z=2$

$3x-3y-4z=11.$

Answer:

$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$C_{11}=-6, C_{21}=17, C_{31}=13$

$C_{12}=14, C_{22}=5, C_{32}=-8$

$C_{13}=-15, C_{23}=9, C_{33}=-1$

$|A|=-6+28+45=67$

$\therefore A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

Now the equations are:

$x+2y-3z=-4$

$2x+3y+2z=2$

$3x-3y-4z=11$

My be written as, $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$\Rightarrow AX = B$

$\Rightarrow X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

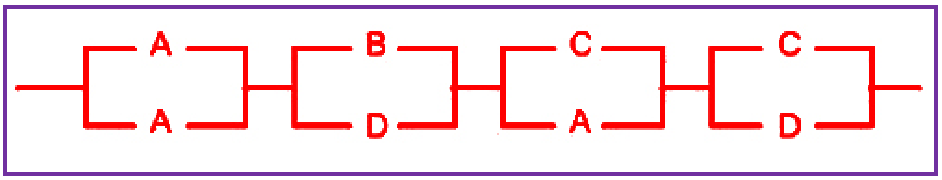
$x = 3, y = -2, z = 1$

Question: 3 [5 × 2=10]

a. i. Show that the second degree equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ represents a pair of straight lines. **

ii. Find the equation of the individual lines and their point of intersection. **

b. i. Write down the Boolean expression corresponding to the switching circuit given below



Answer:

$(A+A)(B+D)(C+A)(C+D)$

$\Rightarrow A(C+A)(B+D)(C+D)$

$\Rightarrow A(C+A)(B+D)(C+D)$

$\Rightarrow (AC+A)(BC+D)$ Distributive law

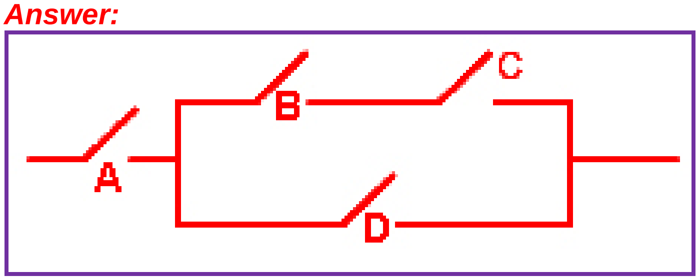
$\Rightarrow A(C+1)(BC+D)$

$\Rightarrow A.(BC+D)$

$\therefore C+1=1$

ii. Simplify the expression and construct the switching circuit for the simplified expression.

Answer:



Question: 4 [5 × 2=10]

a. Solve for x: $\tan^{-1}(x-1) + \tan^{-1}x(x+1) = \tan^{-1}3x.$

Answer:

Given : $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$

$\Rightarrow \tan^{-1}(x+1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$

$$\Rightarrow \tan^{-1}\frac{x-1+x+1}{1-x^2+1} = \tan^{-1}\frac{3x-x}{1+3x^2}$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$$

$$\Rightarrow x\left[1+3x^2-2+x^2\right]=0$$

$$\Rightarrow x = 0 \text{ or } 4x^2-1=0$$

$$\Rightarrow x = \pm\frac{1}{2}$$

b. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Answer:

Given: $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Let, $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$\therefore y = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$$

$$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\tan\frac{\theta}{2}\right].$$

$$y = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\left(1+x^2\right)}$$

Question: 5 [$5 \times 2=10$]

a. Use Lagrange's mean value theorem to determine a point P on the curve $y = \sqrt{x-2}$ defined in the interval $[2,\ 3]$, where the tangent is parallel to the chord joining the end points on the curve.

Answer:

$y = \sqrt{x-2}$ in $[2,3]$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x-1}}$ which exists

i. y is continuous in [2,3]
ii. y is differential in [2,3]

$$\therefore \frac{1}{2\sqrt{x-2}} = \frac{1-0}{1}$$

$$\frac{1}{2} = \sqrt{x-2}$$

$$\frac{1}{4} = x-2$$

$$x = 2 + \frac{1}{4} = \frac{9}{4}\,\text{Î}\,(2,3)$$

$$\therefore y = \sqrt{\frac{9}{4}-2} - \frac{1}{2}$$

$$\therefore \text{Required point P is} \left(\frac{9}{4}, \frac{1}{2}\right).$$

b. An open box with a square base is to be made out of a given quantity of cardboard whose area is c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Answer:

Given : Surface Area of open box $= c^2$

$2bh + 2hl + lb = c^2$

But base is square.

$$\therefore 2ah + 2ah + a^2 = c^2$$

$$\therefore a^2 + 4ah = c^2$$

$$\therefore h = \frac{c^2 - a^2}{4a} \text{....(i)}$$

Now, Volume $= a^2h$

$V = a^2h$

$$\text{or} \quad V = a^2\left(\frac{c^2 - a^2}{4a}\right) = \frac{1}{4}\left(ac^2 - a^3\right)$$

$$\therefore \frac{dV}{da} = \frac{1}{4}\left(c^2 - 3a^2\right) = 0$$

$$\Rightarrow a = \frac{c}{\sqrt{3}}$$

$$\frac{d^2V}{da^2} = \frac{1}{4}(-6a) = \text{-ve}$$

$$\therefore \text{V is maximum at } a = \frac{c}{\sqrt{3}}$$

$$\therefore \text{maxi. Volume} = a^2h$$

$$= \frac{1}{4}\left(ac^2 - a^3\right)$$

$$= \frac{1}{4}\left(\frac{c^3}{\sqrt{3}} - \frac{c^3}{3\sqrt{3}}\right) = \frac{c^3}{4}\left[\frac{2}{3\sqrt{3}}\right]$$

$$= \frac{c^3}{6\sqrt{3}}.$$

Question: 6 [5 × 2=10]

a. Evaluate $\int_0^9 f(x)dx$, where $f(x)$ is defined by $f(x)=\begin{cases}\sin x; \text{if } 0\le x\le \frac{\pi}{2}\\ 1; \frac{\pi}{2}\le x\le 5\\ e^{x-5}; 5\le x\le 9\end{cases}$

Answer:

$$\int_0^9 f(x)dx=\int_0^{\frac{\pi}{2}}\sin x dx+\int_{\frac{\pi}{2}}^{5}dx+\int_5^9 e^{x-5}dx$$

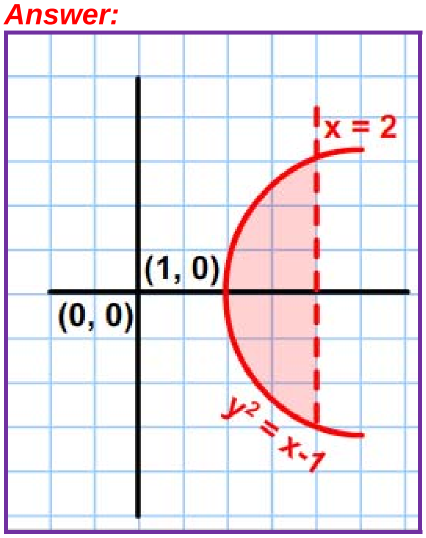
$$=[-\cos x]_0^{\frac{\pi}{2}}+[x]_{\frac{\pi}{2}}^{5}+\left[e^{x-5}\right]_5^9$$

$$=[1]+\left[5-\frac{\pi}{2}\right]+\left[e^4-1\right]$$

$$=\left(e^4+5-\frac{\pi}{2}\right).$$

b. Draw a rough sketch of the curve $y^2+1=x,\ x\le 2$. Find the area enclosed by the curve and the line $x=2$.

Answer:



Given curve: $y^2+1=x,\ x\le 2$

$\Rightarrow y^2=x-1$

$$\text{Required Area} = 2\int_1^2 \sqrt{x-1}\,dx$$

$$= \frac{2\left[(x-1)^{\frac{3}{2}}\right]_1^2}{\frac{3}{2}}$$

$$= 2\times\frac{2}{3}\left[(1)^{\frac{3}{2}}\right]$$

Question: 7

[5 × 2=10]

The data for marks in physics and History obtained by ten students are given below:

Marks in physics	15	12	8	8	7	7	7	6	5	3
Marks in history	10	25	17	11	13	17	20	13	9	15

Using in data:

a. Compute Karl Pearson's coefficient of correlation between the marks in physics and History obtained by the 10 students.

Answer:

Given marks are:

Physics (x)	History (y)	xy	x^2	y^2
15	10	150	225	100
12	25	300	144	625
8	17	136	64	289
8	11	88	64	121
7	13	91	49	169
7	17	119	49	289
7	20	140	49	400
6	13	78	36	169
5	9	45	25	81
3	15	45	9	225
78	150	1192	714	2468

Karl Pearson's coefficient:

$$r = \frac{\frac{1}{n}\left[\sum xy - \frac{\sum x \sum y}{n}\right]}{\sqrt{\left\{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2\right\}\left\{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2\right\}}}$$

$$= \frac{1192 - \frac{78\times 150}{10}}{\sqrt{(714-608.4)(2468-2250)}}$$

$$= \frac{22}{\sqrt{105.6 \times 218}}$$

$$= \frac{22}{151.7} = \pm .14.$$

b. i. Find the line of regression in which physics is taken as the independent variable.

Answer:

Line of y on x

$$y - \bar{y}\left(b_{yx} y - \bar{x}\right)$$

$$\text{Here } b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{1192 - \frac{78 \times 150}{10}}{714 - \frac{78 \times 78}{10}}$$

$$= \frac{22}{105.6} = \frac{5}{24}$$

$$\bar{x} = \frac{\sum x}{n} = 7.8$$

$$\bar{y} = \frac{\sum y}{n} = 15$$

$\therefore$ Equation corese of regression is

$$\text{(i)}\, y - 15 = \frac{5}{24}(x - 7.8)$$

i. A candidate had scored 10 marks in Physics but was absent from the History test. Estimate his probable score for the latter lest.

Answer:

$$y - 15 = \frac{5}{24}(2.2)$$

$$y = \frac{11}{24} + 15 = \frac{11 + 360}{24} = \frac{371}{24} = 15.4\, \textit{(Approx.)}$$

***Section B** (Question numbers 8 to 10)*

Question: 8 [5 × 2=10]

a. There are 3 urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and 1 black ball?

Answer:

RRB + BRR + RBR

Three cases are possible.

$$\left(\frac{4}{7}\times\frac{5}{9}\times\frac{4}{8}\right)+\left(\frac{3}{7}\times\frac{5}{9}\times\frac{4}{8}\right)+\left(\frac{4}{7}+\frac{4}{9}+\frac{4}{8}\right)$$

$$=\frac{80+60+64}{504}=\frac{204}{504}=\frac{102}{252}=\frac{51}{126}$$

$$y=\frac{x}{2}$$

b. The probability that a teacher will give an unannounced test during any class meeting is $\frac{1}{5}$. If student is absent twice, find the probability that the student will miss at least one test.

Answer:

$$p=\frac{1}{5}, q=\frac{4}{5}, n=2$$

$$\text{Required probability} = {}^{2}C_{1}\left(\frac{1}{5}\right)^{1}\left(\frac{4}{5}\right)+{}^{2}C_{2}\left(\frac{1}{5}\right)^{2}$$

$$=2\times\frac{4}{25}+\frac{1}{25}$$

$$=\frac{8}{25}+\frac{1}{25}=\frac{9}{25}.$$

Question: 9 [5 × 2=10]

a. If the ratio $\frac{z-i}{z-1}$ is purely imaginary, prove that the point z lies on the circle whose centre is the point $\frac{1}{2}(1+i)$ and radius is $\frac{1}{\sqrt{2}}$.

Answer:

$$\frac{z-i}{z-1}=\frac{x+iy-i}{x+iy-1}=\frac{x+i(y-1)}{(x-1)+iy}$$

$$\Rightarrow \frac{\{x+(y-1)\}\{(x-1)-iy\}}{\{(x-1)+iy\}\{(x-1)-iy\}}$$

$$\frac{x(x-1)+y(y-1)+i\{(y-1)(x-1)-xy\}}{(x-1)^2+y^2}$$

$$\Rightarrow \frac{x^2+y^2-x-y}{x^2+y^2-2x+1}=0 \quad \text{Q it is purely imaginary}$$

$$\Rightarrow x^2+y^2-x-y=0$$

This is equation of a circle whose centre is $\left(\frac{1}{2},\frac{1}{2}\right)$ i.e., $\frac{1}{2}(1+i)$ and radius is

$$\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-0}$$

i.e., $\frac{1}{\sqrt{2}}$.

b. Solve : $\left(x^2+y^2\right)dx-2xy\,dy=0$, given that $y=0$, when $x=1$.

Answer:

Given: $\left(x^2+y^2\right)dx-2xy\,dy=0$

$\&\ y=0$ when $x=1$

Given equation may be written as-

$\therefore y = vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Given equation becomes-

$$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2x^2v}$$

$$v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1-v^2}{2v}$$

$$\Rightarrow \frac{2v}{1-v^2}dv=\frac{dx}{x}$$

Integrating both the sides

$$\int\frac{2v}{1-v^2}dv=\int\frac{dx}{x}$$

$$-\log|1-v^2|=\log x+c$$

$$\Rightarrow c=0 \qquad \text{Q}y=0 \text{ when } x=1$$

$$\therefore \frac{1}{1-\frac{y^2}{x^2}}=x$$

$$\Rightarrow x=x^2-y^2$$

Question: 10 [5 × 2=10]

a. Find the coordinates of the point where the line joining the points $(1,-2,\ 3)$ and $(2,-1,\ 5)$ cuts the plane $x-2y+3z=19$. Hence, find the distance of this point from the point $(5,\ 4,\ 1)$.

Answer:

Equation of line passing through $(1,-2,3)$ & $(2,-1,5)$ is $\frac{x-1}{1}=\frac{y+2}{1}=\frac{z-3}{2}=k$

Let any point P on this line be $(k+1,k-2,2k+3)$

This point P lines on the plane

$x-2y+3z=19$

$\therefore k+1-2k+4+6k+9=19$

$5k=19-14=5$

$k=1$

$\therefore$ Point be $(2,-1,\ 5)$.

Distance $=\sqrt{(5-2)^2+(4+1)^2+(1-5)^2}$

$=\sqrt{9+25+16}$

$=\sqrt{50}=5\sqrt{2}$.

b. If $A(-1,\ 4,-3)$ is one end of a diameter AB of the sphere $x^2+y^2+z^2-2y+2z-15=0$ then find the coordinates of the other end point B. **

Section C *(Question numbers 11 to 15)*

Question: 11

[5 × 2=10]

a. A small industrial concern used three raw materials A, B and C in its manufacturing process. The prices of the materials was as shown below:

commodity	Price in Rs. In the year 1995	Price in Rs. In the year 2005
A	4	5
B	60	57
C	36	42

Using 1995 as the base year, calculate a simple aggregate price index for 2005.

Answer:

$$\text{Price index} = \frac{\Sigma p_2}{\Sigma p_1}\times 100 = \frac{104}{100}\times 100 = 104.$$

b. Coded monthly sales figures of a particular brand of T.V. for 18 months commencing January 1, 2005 are as follows:

YEAR	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
2005	18	16	23	27	28	19	31	29	35	27	28	24
2006	24	28	29	30	29	22						

Calculate six monthly moving averages and display these and the original figures on the same graph using the same axes for both. **

Question: 12

[5 × 2=10]

a. Find $\vec{a}.\vec{b}$ if $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 8$.

Answer:

$|a\times b| = |a||b|\sin\theta$

$\therefore \sin\theta = \frac{8}{2\times 5} = \frac{4}{5}$

$\therefore \cos\theta = \frac{3}{5}$

$\therefore a.b = |a||b|\cos\theta$

$= 2\times 5\times\frac{3}{5} = 6.$

b. Given $\vec{a} = i - 2j + k, \vec{b} = 2i + j + k$ and $\vec{c} = i + 2j - k$, Find : $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$.

Answer:

$$a \times (b \times c) = (i - 2j + k) \times \{(2i + j + k) X (i + 2j - k)\}$$

$$= (a.c)b - (a.b)c$$

$$= (1 - 4 - 1)(2i - j + k) - (2 - 2 + 1)(i + 2j - k)$$

$$= -8i - 4j - 4k - i - 2j + k$$

$$= -9i - 6j - 3k.$$

Question: 13

[5 × 2=10]

a. The baker's gain on a certain bill due 6 months hence is Rs.100, the rate of interest being 10% per annum. Find the face value of the bill.

Answer:

Banker's gain is interest on T.D.

$$100 = \text{T.D.} X \frac{6}{2} \times \frac{10}{100}$$

$$\text{T.D.} = \text{Rs. } 2000$$

B.D. is interest on face value.

$$\therefore 2100 = \text{Face value} \frac{6}{2} \times \frac{10}{100}$$

$$\text{Face value} = 2100 \times 20$$

$$= \text{Rs. } 42000.$$

b. Mr. Aggarwal buys a house at Rs. 30, 00,000 for which he agrees to make equal payments at the end of each year for 10 years. If money is worth 10% p.a., find the amount of each instalment. [Take $(1.1) = 0.3855$]

Answer:

$$v = \frac{A}{r}\left[1 - (1 + r)^{-n}\right]$$

$$\text{Given: } r = \frac{10}{100} = .1$$

$$\therefore 30{,}00{,}000 = \frac{A}{.1}\left[1 - (1.1)^{-10}\right]$$

$$3{,}00{,}000 = A[1 - .3855] \quad \because (1.1)^{-10} = .3855 \text{ Given}$$

$$\therefore A = \frac{3{,}00{,}000}{.6145}$$

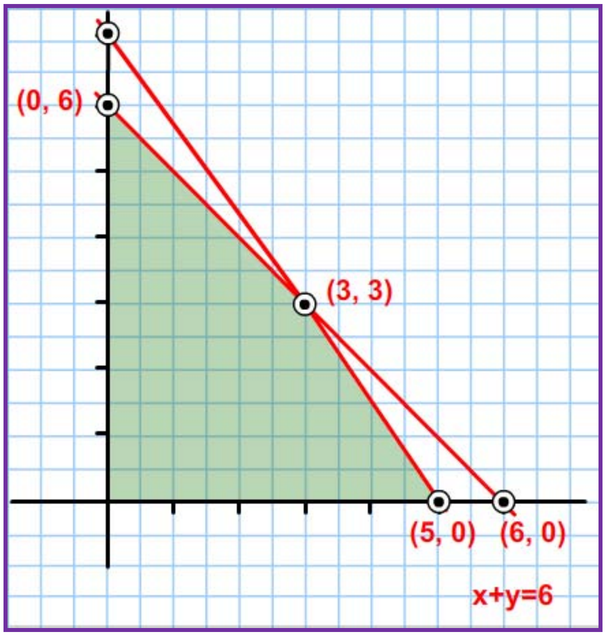
$$= \text{Rs. } 488201.79.$$

Question: 14

[5 × 2=10]

a. A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. Machines A and B can work at most for 18 hours per day respectively. He earns a profit of Rs. 30 per trunk on the first type of trunk and Rs.25 per trunk on the second type. Formulate a Linear programming problem to find out how many trunks of each type he must make each day to maximize his profit.

Answer:



Let x unit of A type & y unit of B type be produced.

$\therefore z = 30x + 25y$

Under the following restrictions

$3x + 3y \le 18$

$3x + 2y \le 15$

$x \ge 0, y \ge 0$

Corner points of shaded portion are

$(0,0), (0,6), (5,0), (3,3)$.

z is maximum at $(3,3)$.

$\therefore x = 3 \,\&\, y = 3$.

b. The average cost function associated with producing and marketing x units an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find;

i. The total cost function and marginal coat function.
ii. The range of values of the output x for which AC is increasing.

Answer:

Given : $AC = 2x - 11 + \frac{50}{x}$

i. Total cost function:

$T.C = 2x^2 - 11x + 50$

$M.C = \frac{d}{dx}(T.C.) = 4x - 11$.

ii. $\frac{d}{dx}(AC) = 2 - \frac{50}{x^2} > 0$

$2 > \frac{50}{x^2}$.

$x^2 > 25$

$x > 5, x > -5$

Question: 15

[5]

a. Eight coins are thrown simultaneously.

i. Show that the probability of getting at 6 heads is $\frac{37}{256}$.

ii. What is the probability of getting at least 3 heads?

Answer:

i. $p=\frac{1}{2}, q=\frac{1}{2}, n=8$

Probability (at lest 6 heads):

$$={}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2+{}^8C_7\left(\frac{1}{2}\right)^8+{}^8C_8\left(\frac{1}{2}\right)^8$$

$$=\left[\frac{8\times 7}{1\times 2}+8+1\right]\left(\frac{1}{2}\right)^8$$

$$=37\times\frac{1}{256}=\frac{37}{256}.$$

ii. Probability (at lest three heads)

$$=i-{}^8C_0\left(\frac{1}{2}\right)^8-{}^8C_1\left(\frac{1}{2}\right)^8+{}^8C_2\left(\frac{1}{2}\right)^8$$

$$=1-\frac{1}{2^8}[1+8+28]=1-\frac{37}{256}$$

$$=\frac{256-37}{256}=\frac{219}{256}.$$

b. A class consists of 50 students out of which there are 10 girls. In the class 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be a rank holder, what is the probability that the student selected is a girl ?

Answer:

$$P\left(\frac{G}{R}\right)=\frac{P\left(\frac{R}{G}\right).P(G)}{P\left(\frac{R}{G}\right).P(G)+P\left(\frac{R}{B}\right).P(B)}=\frac{\frac{1}{5}\times\frac{1}{5}}{\frac{1}{5}\times\frac{1}{5}+\frac{4}{5}\times\frac{5}{40}}$$

$$=\frac{\frac{1}{25}}{\frac{1}{25}+\frac{1}{10}}=\frac{\frac{1}{25}}{\frac{2+5}{50}}=\frac{2}{7}.$$

*** Out of syllabus. Answer should be provided up on request*