2012

Section A (Compulsory) *(Question numbers 1 to 7)*

Question: 1 [3x10 = 30]

i. Solve for x and y if $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} 2x \\ 3y \end{pmatrix} = 3\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

Answer:

$$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + \begin{pmatrix} 4x \\ 6y \end{pmatrix} = \begin{pmatrix} 21 \\ -9 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x^2 + 4x \\ y^2 + 6y \end{pmatrix} = \begin{pmatrix} 21 \\ -9 \end{pmatrix}$$

$x^2 + 4x = 21$	$y^2 + 6y = -9$
$x^2 + 4x - 21 = 0$	$y^2 + 6y + 9 = 0$
$x^2 + 7x - 3x - 21 = 0$	$y^2 + 3y + 3y + 9 = 0$
$x(x+7) - 3(x-3) = 0$	$(y+3)(y+3) = 0$
$x = -7$ or $x = 3$	$y = -3$

ii. Prove that $\sec^2(\tan^{-1} 2) + \text{cosec}^2 (\cot^{-1}) = 15$

Answer:

Given $\sec^2(\tan^{-1} 2) + \text{cosec}^2 (\cot^{-1}) = 15$
Let $\tan^{-1} 2 = \theta$, $\cot^{-1} 3 = \theta$
L.H.S = $\sec^2 \theta + \text{cosec}^2 \theta$
= $(1 + \tan^2 \theta) + (1 + \cot^2 \theta)$
= 1+4+1+9
= 15 = R.H.S

iii. Find the equation of the hyperbola whose Transverse and Conjugate axes are the x and y axes respectively, given that the length of conjugate axis is 5 and distance between the foci is 13.

Answer:

Equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given: $2b = 5 \Rightarrow b = \frac{5}{2}, 2ae = 13 \Rightarrow \frac{13}{2}$

We know that $b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = a^2e^2 - a^2$

$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2$

$a^2 = \frac{169}{4} - \frac{25}{4} = \frac{144}{4} = 36$

$\Rightarrow a = 6, b = \frac{5}{2}$

Equation of Hyperbola

$$\frac{x^2}{36} - \frac{y^2}{\frac{25}{4}}$$

iv. Find the equations of the two regression lines, 4x + 3y + 7 = 0 and 3x + 4y + 8 = 0, find:

a. Mean of x and y.
b. Regression coefficients
c. Coefficient of correlation.

Answer:

4x + 3y =- 7 and 3x + 4y =- 8

Solving equation (1) and (2), we get

$$x = -\frac{4}{7}, y = -\frac{11}{7}$$

$$\bar{x} = -\frac{4}{7}, \bar{y} = -\frac{11}{7}$$

$$x = -\frac{3}{4}y - \frac{7}{4}, \text{ we get } b_{xy} = -\frac{3}{4}$$

$$\text{From equation(2)}, y = -\frac{3}{4}x - \frac{8}{4}, \text{ we get } b_{yx} = -\frac{3}{4}$$

$$\therefore \text{Regression coefficient } r = \sqrt{b_{yx}\, b_{xy}}$$

$$= \sqrt{\left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right)} = -\left(\frac{3}{4}\right)$$

v. Evaluate : $\int e^x(\tan x + \log\sec x)dx$

Answer:

$$\int e^x(\tan x + \log\sec x)dx$$

$$= \int e^x \tan x\, dx + \int \underset{\text{I}}{\log(\sec x)} . \underset{\text{II}}{e^x}\, dx$$

$$= \int e^x \tan x\, dx + \log(\sec x)\int e^x\, dx - \int\left\{\frac{d}{dx}\log(\sec x). \int e^x\, dx\right\}dx$$

$$= \int e^x \tan x\, dx + \log(\sec x).e^x - \int \frac{1.\sec x.\tan x}{\sec x}.e^x\, dx + c$$

$$= \int e^x \tan x\, dx + \log(\sec x).e^x - = \int e^x \tan x\, dx + c$$

$$= \log(\sec x).e^x + c$$

vi. Evaluate : $\lim_{x \to \frac{x}{2}} (\cos.\log\tan x)$

Answer:

$$\text{R.H.L} = \lim_{h \to 0} \cos\log\tan\left(\frac{\pi}{2} + h\right)$$

$$= \lim_{h \to 0} \cos \log(-\cot h)$$

$$= \cos\log(-\infty) = \text{n.d}$$

$$\text{L.H.L} = \lim_{h \to 0} \cos \log \tan\left(\frac{\pi}{2} - h\right)$$

$$= \lim_{h \to 0} \cos \log(\cot h)$$

$$= \lim_{h \to 0} \cos[\log \cos h - \log \sin]$$

$$= \cos\left[(\log \cos 0) - \lim_{h \to 0} \log \sin h\right]$$

$$= \cos[0 - \log \sin 0]$$

$$= \cos(-\infty)$$

$\therefore$ Limit does not exist.

vii. Find the locus of the complex number, $Z = x + iy$ given $\left|\frac{x + iy - 2i}{x + iy + 2i}\right| = \sqrt{2}$

Answer:

Given: $\left|\frac{x + iy - 2i}{x + iy + 2i}\right| = \sqrt{2}$

$$\Rightarrow \left|\frac{x + i(y - 2)}{x + i(y + 2)}\right| = \sqrt{2}$$

$$\Rightarrow \frac{\sqrt{x^2 + (y-2)^2}}{\sqrt{x^2 + (y+2)^2}} = \sqrt{2}$$

On squaring both sides

$2[x^2(y+2)^2] = x^2(y-2)^2$

$\Rightarrow 2[x^2 + y^2 + 4y + 4] = x^2 + y^2 - 4y + 4$

$\Rightarrow x^2 + y^2 + 12y + 4 = 0$

$(x-0)2 + [y-(-6)^2] - 32 = 0$

$\Rightarrow (x-0)^2 + [y-(-6)^2] = \left(4\sqrt{2}\right)^2$

Which is the equation of circle with centre (0,-6) and $\left(4\sqrt{2}\right)$.

viii. Evaluate : $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$

Answer:

Let $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$ ....(1)

$$I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-(3-x)}\sqrt{3-x} + \sqrt{x}} dx$$

$= \int_1^2 \left(\frac{\sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}}\right) dx$ ....(2)

On adding eq. (1) and (2), we get

$$2I = \int_1^2 \left(\frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} + \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \right) dx$$

$$= \int_1^2 \left(\frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}} \right) dx$$

$$= \int_1^2 1 dx = [x]_1^2 = 2 - 1 = 1$$

$$I = \frac{1}{2}$$

ix. Three persons A,B and C shoot to hit a target. If in trails, A hits the target 4 times in 5 shots, B hits 3 times in 4 shots and C hits 2 times in 3 trials. Find the probability that:

a. Exactly two persons hit the target.

Answer:

$$\text{Let } P(A) = \frac{4}{5}, \quad P(B) = \frac{3}{4} \quad P(C) = \frac{2}{3}$$

$$P(\bar{A}) = 1 - \frac{4}{5} = \frac{1}{5} \quad P(\bar{B}) = 1 - \frac{3}{4} = \frac{1}{4} \quad P(\bar{C}) 1 - = \frac{2}{3} = \frac{1}{3}$$

$$= P(AB\bar{C}) + P(BC\bar{A}) + P(CA\bar{B})$$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{3} \times \frac{4}{5} \times \frac{1}{4}$$

$$= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{60}$$

b. At least two persons hit the target.

Answer:

P(at least two persons hit the target)

$$= P(AB\bar{C}) + P(BC\bar{A}) + P(CA\bar{B}) + P(ABC)$$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5} + \frac{2}{3} \times \frac{4}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} + \frac{24}{60} = \frac{50}{60} + \frac{5}{6}$$

x. Solve the differential equation: $(xy^2+x)dx+(x^2y + y)dy = 0$

Answer:

Given: $(xy^2+x)dx+(x^2y + y)dy = 0$
Or $x(y^2+1)dx+y(x^2+1)dy = 0$
Or $x(y^2+1)dx = -y(x^2+1)dy$

$$\text{or } \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(y^2+1\right) = c$$

$$\text{or } \frac{x}{x^2+1}dx = \frac{y}{y^2+1}dy$$

$$\int \frac{x}{x^2+1}dx = -\int \frac{y}{y^2+1}dy$$

or $$\frac{1}{2}\int \frac{2x}{x^2+1}dx = -\int \frac{2y}{y^2+1}dy$$

or $$\frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2+1\right) = c$$

Or $\log(x^2+1)+\log(y^2+1) = 2c$
Or $\log(x^2+1)\ (y^2+1) = 2c$
Or $(x^2+1)\ (y^2+1) = e^{2c}$

Question: 2

[5+5=10]

a. Using properties of determinants, prove that:

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Answer:

R.H.S = Applying $R_2 \to R_2 \to 2R_1$ and $R_3 \to R_3 - 3R_1$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

Expanding by first column
$=a(a(7a+3b) - 3a(2a+b))$
$=a(7a^2+3ab-6a^2-3ab)$
$=a \times a^2 = a^3$ = R.H.S

b. Find the product of the matrices A and B where:

$$A = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$ hence, solve the following equations by

Matrix method:
$x+y+2z = 1$
$3x+2y+z = 7$
$2x+y+3z = 2$

Answer:

$$\text{Let } AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$\Rightarrow BX = C$

$\Rightarrow X = B^{-1} C$

$A.B = 4I_3$

$$\Rightarrow \left(\frac{A}{4}\right).B = I_3$$

$$\Rightarrow B^{-1} = \frac{1}{4}A$$

$$= \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Question: 3 [5+5=10]

a. Prove that: $\cos^{-1} \cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$

Answer:

$$\text{L.H.S} = \cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5}$$

$$\text{Let } \cos^{-1}\left(\frac{63}{65}\right) = \theta \Rightarrow \cos\theta = \frac{63}{65}$$

$$BC = \sqrt{AC^2 - AB^2}$$

$$= \sqrt{65^2 - 63^2} = \sqrt{4225 - 3969} = \sqrt{256} = 16$$

$$\Rightarrow \theta \tan^{-1}\frac{16}{63}$$

$$\cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{16}{63}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$$

$$= \tan^{-1}\left(\frac{16}{63}\right) + \tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right) \quad \left[\because 2\tan^{-1}x = \tan^{-1}\left[\frac{2x}{1-x^2}\right]\right]$$

$$= \tan^{-1}\left(\frac{16}{63}\right) + \tan^{-1}\left(\frac{2}{5}\times\frac{25}{24}\right) = \tan^{-1}\left(\frac{16}{63}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{\frac{16}{63}+\frac{5}{12}}{1-\frac{16}{63}\times\frac{5}{12}}\right) = \tan^{-1}\left(\frac{\frac{16\times 12+5\times 63}{63\times 12}}{\frac{63\times 12+5\times 63}{63\times 12}}\right)$$

$$= \tan^{-1}\left(\frac{507}{676}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

Let $\phi = \tan^{-1}\left(\frac{3}{4}\right)$

In right ΔABC, AC $= \sqrt{AB^2 + BC^2}$

$= \sqrt{32+4^2} = \sqrt{25} = 5$

$\sin\varphi = \frac{BC}{AC} = \frac{3}{5}$

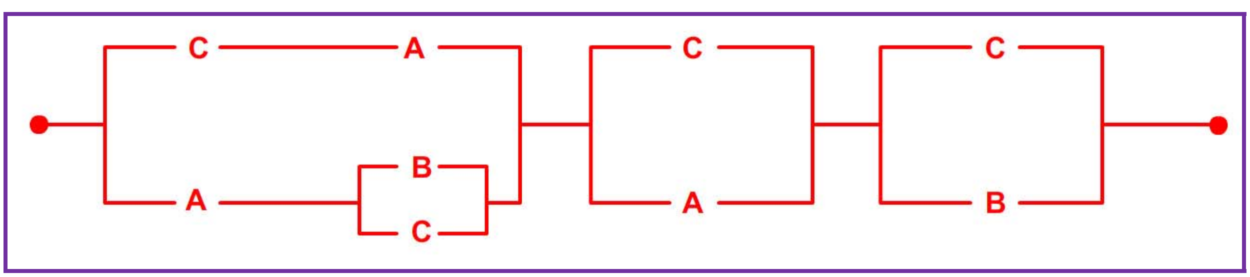
$\varphi = \sin^{-1}\frac{3}{5}$

$\tan^{-1}\left(\frac{3}{4}\right) = \sin^{-1}\frac{3}{5} = \text{R.H.S}$

b. i. write the Boolean expression corresponding to the circuit given below:

Answer:

ii. Simplify the expression using laws of Boolean Algebra and construct the simplified circuit.



Answer:
(C.A+A.(B.C).(C+A).(C+B)
=(A.C+A.(B+C)).(C+A).(C+B)
=(A.C+A.B+A.C).(C+A).(C+B)
=(A.(C+C)+ A.B).(C+A).(C+B)
=(A.C+A.B).(C+A).(C+B)
=A.(C+B).(C+A).(C+B)
=A.(C+B). (C+B). (C+A)

=A.(C+B). (C+A) [As C+BC=C]
=A.(C+CA+BA)   (As C+CA = C)
= A. (C+BA)
= AC + AAB = AC + AB = A (C+B)
Simplified circuit:
PICTURE

Question: 4 [5+5=10]

a. Verify Rolle's theorem for the function:

$f(x) = \log \left\{\frac{x^2+ab}{(a+b)}\right\}$ in the interval [a,b] where 0 ϵ[a,b]

Answer:

We have $f(x)=\log\left\{\frac{x^2+ab}{x(a+b)}\right\}$

= $\log(x^2+ab)-\log x - \log(a+b)$

Since logarithmic function is differentiable and so continuous on its domain. Therefore f(x) is continuous on [a,b] and differentiable on (a,b).

Also, $f(a)=\log\left\{\frac{a^2+ab}{a(a+b)}\right\}=\log 1=0$

And $f(b)=\log\left\{\frac{b^2+ab}{b(a+b)}\right\}=\log 1=0$

$\therefore$ f(a) = f(b)

Thus all three conditions of Rolle's theorem are satisfied.

Now, we have to show there exists $c\in$ (a,b) such that f'(c) = 0

We have $f(x) = \log(x^2+ab)-\log x - \log(a+b)$

$$f'(x)=\frac{2x}{x^2+ab}-\frac{1}{x}=\frac{x^2-ab}{x(x^2+ab)}$$

$\therefore f'(x)=0$

$$\Rightarrow \frac{x^2-ab}{x(x^2+ab)}=0$$

$\Rightarrow x^2=ab$

$\Rightarrow x=\sqrt{ab}$

Since $c=\sqrt{ab}\in(a,b)$ such thar $f'(c)=0$

Hence, Rolle's theorem is verified.

b. Find the equation of the ellipse with its centre at (4,-1) focus at (1,-1) and given that it passes through (8,0).

Answer:

Equation of ellipse with its major axis parallel to x axis and having centre(h,k)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad ....(1)$$

$CF'=ae$

$$\Rightarrow \sqrt{(4-1)^2+(-1+1)^2}=ae$$

$\Rightarrow$ ae = 3 .....(2)
Ellipse(1) also passes through (8,0) and centre (4,-1)

$$\therefore \frac{(8-4)^2}{a^2}+\frac{(0+1)^2}{b^2}=1$$

$$\Rightarrow \frac{16}{a^2}+\frac{1}{b^2}=1$$

From eq.(3)

$$\frac{16}{a^2}+\frac{1}{a^2-9}=1$$

$$\frac{16}{t}+\frac{1}{t-9}=\frac{1}{1}$$

$$\frac{16t-144+t}{t(t-9)}=1$$

$$\Rightarrow t^2-9t=17t-144$$

$$\Rightarrow t^2-26t+144=0$$

$$\Rightarrow t^2-18t-8t+144=0$$

$$\Rightarrow t(t-18)-8(t-18)=0$$

$$\Rightarrow (t-18)(t-18)=0$$

$$\Rightarrow =18 \text{ or } t=8$$

$$\Rightarrow a^2=18 \text{ or } a^2=8$$

If a^2 = 8, b^2 = 8-9 =-1(which is not possible)
$\therefore a^2$ = 18, b^2 = 18-9 = 9
Equation of ellipse

$$\frac{(x-4)^2}{18}+\frac{(y+1)^2}{9}=1$$

Question: 5 [5+5=10]

a. If $e^y(x+1)$ = 1, then show that:

$$\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$$

Answer:
Given $e^y(x+1)$ = 1
Taking logarithm of both sides
$\Rightarrow \log_e[e^y(x+1)] = \log e\ 1$
$\Rightarrow \log_e ey + \log e(x+1) = 0$
$\Rightarrow y = -\log_e (x+1)$
On differentiating

$$y=\frac{dy}{dx}=-\frac{1}{(x+1)}\times\frac{d}{dx}(x+1)=-\frac{1}{(x+1)}\text{...(1)}$$

Again differentiating w.r.t.x

$$\frac{d^2y}{dx^2}=+\frac{1}{(x+1)^2}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

b. A printed page is to have a total are of 90 sq. cm with a margin of 1cm at the top and on each side and margin of 1.5cm at the bottom. What should be the dimensions of the page so that the printed area will be maximum?

Answer:

Let the length of page be x cm and breadth be y cm. according to problem,

x.y = 80 sq. cm.

length of printed material = (x-2.5) cm

breadth of printed material = (y-2)cm

A=(x-2.5)(y-2)

= xy – 2x – 2.5y + 5

$$A = 80 - 2x - 2.5 \times \frac{80}{x} + 5$$

$$= 85 - 2x \times \frac{200}{x}$$

$$\frac{dA}{dx} = 0$$

$$\Rightarrow -2 + \frac{200}{x^2} = 0 \quad \Rightarrow \frac{200}{x^2} = 2$$

$$\Rightarrow x^2 = 100$$

$$\Rightarrow x = 10$$

$$\frac{d^2A}{dx^2} = -\frac{200 \times 2}{x^3} = -\frac{400}{x^3}$$

$$\left(\frac{d^2A}{dx^2}\right)_{(x-10)} = -\frac{400}{10^3} = \frac{4}{10} = -\frac{2}{5} = \text{negative}$$

$\therefore$ A will be maximum when x = 10cm.

x.y = 80

$\Rightarrow$ 10.y = 80

$\Rightarrow$ y = 8 cm

Dimensions: length of paper = 10cm

Breadth of paper = 8 cm

Question: 6 [5+5=10]

a. Evaluate: $\int \frac{dx}{x\{6(\log x)^2 + 7\log x + 2\}}$

Answer:

$$\text{Given} : I = \int \frac{dx}{x\{6(\log x)^2 + 7\log x + 2\}} \text{put} \log x = t \Rightarrow \frac{1}{x}dx = dt$$

$$I = \int \frac{dx}{x6t^2 + 7t + 2}dt$$

$$= \frac{1}{6}\int \frac{1}{6t^2 + \frac{7}{6}t + \frac{1}{3}}dt$$

$$= \frac{1}{6}\int \frac{1}{\left(t+\frac{7}{12}\right)^2 + \frac{1}{3} - \frac{49}{144}} dt$$

$$= \frac{1}{6}\int \frac{1}{\left(t+\frac{7}{12}\right)^2 + \left(\frac{1}{12}\right)^2} dt$$

$$= \frac{1}{6} \cdot \frac{1}{2\left(\frac{1}{12}\right)} \log\left|\frac{\left(t+\frac{7}{12}\right) - \left(\frac{1}{12}\right)}{\left(t+\frac{7}{12}\right) + \left(\frac{1}{12}\right)}\right| + c$$

$$= \log\left|\frac{6t+3}{6t+4}\right| + c$$

$$= \log\left|\frac{6\log x+3}{6\log x+4}\right| + c$$

b. Find the area of the region bounded by the curve $x = 4y - y^2$ and the y-axis. [5]

Answer:

Given: $x = 4y-y^2$

$x = -(y^2-4y+4-4)$

$= -((y-2)^2-4)$

$=-(y-2)^2+4$

$\Rightarrow (y-2)^2=4-x=-(x-4)$

Let y-2 = y, x-4 = x

$\Rightarrow y^2 = -x$

Which represents a parabola whose axis is parallel to x axis

Vertex x = 0, y = 0

$\Rightarrow x - 4 = 0, y - 2 = 0$

$\Rightarrow x = 4, y = 2$

It cuts y axis whn x = 0

$\Rightarrow (y-2)^2 = 4 - 0 = 4$

$\Rightarrow y - 2 = \pm 2$

$\Rightarrow y = \Rightarrow 2 + 2$

$\Rightarrow y = 4, y = 0$

Area bounded between y – axis and parabola

$$= \int_0^4 x\,dy$$

$$= \int_0^4 (4y - y^2)\,dy$$

$$= \left[4.\frac{y^2}{2} - \frac{y^3}{3}\right]_0^4$$

$$= \left[\left(2\times 4^2 - \frac{4^3}{3}\right) - 0\right] = \left[32 - \frac{64}{3}\right]$$

$$= \frac{32}{3}\text{sq.units.}$$

Question: 7 [5+5=10]

a. Ten candidates received percentage marks in two subject as follows:

Candidate	A	B	C	D	E	F	G	H	I	J
Mathematics marks	80	88	76	74	68	65	40	43	40	80
Statistics marks	72	84	90	66	54	50	54	38	30	43

Calculate Spearman's rank correlation coefficient and interpret your result.

Answer:

	Mathematics marks(x)	Statistics marks(y)	Rank in x(R_1)	Rank in y(R_2)	D = $R_1 - R_2$	d^2
A	80	72	2.5	3	-.5	0.25
B	88	84	1	2	-1	1
C	76	90	4	1	3	9
D	74	66	5	4	1	1
E	68	54	6	5.5	1.5	.25
F	65	50	7	7	0	0
G	40	54	8.5	5.5	3	9
H	43	38	8	9	-1	1
I	40	30	8.5	10	-1.5	2.25
J	80	43	2.5	8	-5.5	30.25

$\Sigma d^2 = 5.4$

$$R = 1 - 6\left[\sum D^2 + \frac{1}{12}(m_2{}^3 - m_2) + \frac{1}{12}(m_3{}^3 - m_3)\right]$$

$$= 1 - 6\left[54 + \frac{1}{12}\left(2^3 - 2\right) + \frac{1}{12}\left(2^3 - 2\right) + \frac{1}{12}\left(2^3 - 2\right)\right]$$

$$= 1 - \frac{\left[54 + \frac{3}{2}\right]}{10 \times 99} = 1 - \frac{6 \times 55.5}{10 \times 99}$$

$$= 1 - \frac{333}{990} = \frac{990 - 333}{990} = \frac{657}{990} = 0.66$$

Marks in Maths and Statistics have direct correlation of moderate degree i.e increase in marks in Statistics shows increase in marks in Maths.

b. The following results were obtained with respect to two variables x and y:
$\Sigma x = 30$, Σ=42, $\Sigma xy = 199$, $\Sigma x^2 = 184$, $\Sigma y^2 = 318$, n = 6
Find the following:

i. The regression coefficients

Answer:

Given: $\Sigma x = 30$, Σ=42, $\Sigma xy = 199$, $\Sigma x^2 = 184$, $\Sigma y^2 = 318$, n = 6

$$b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{\left(\sum y^2\right)}{n}}$$

$$= \frac{199 - \frac{30 \times 42}{6}}{318 - \frac{42 \times 42}{6}} = \frac{199 - 210}{318 - 294} = -\frac{11}{24}$$

$$b_{yx} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{\left(\sum x^2\right)}{n}}$$

$$= \frac{199 - \frac{30 \times 42}{6}}{184 - \frac{(30)^2}{6}} = \frac{199 - 210}{184 - \frac{900}{6}} = -\frac{11}{34}$$

ii. Correlation coefficient between x and y.

Answer:

Since b_{yx} and b_{xy} are both negative therefore r must also be negative.

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$= -\sqrt{\frac{-11}{24} \times \frac{-11}{34}} = -\frac{-11}{\sqrt{24 \times 34}} = \frac{-11}{\sqrt{316}} = -\frac{11}{28.6} = -0.38$$

iii. Regression equation of y on x.

Answer:

Regression equation of y on x.

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$\bar{x} = \frac{\sum x}{n} = \frac{30}{6} = 5, \quad \bar{y} = \frac{\sum y}{n} = \frac{42}{6} = 7$$

$$b_{yx} = \frac{11}{34}$$

Regression equation of y on x

$$y - 7 = -\frac{11}{34}(x - 5)$$

$\Rightarrow$ 34y-238=-11x+55

$\Rightarrow$11x+34y-293 = 0

iv. The likely value of y when x = 10.

Answer:

When x = 10

$\Rightarrow$ 11 $\times$ 10 + 34y-293 = 0

$\Rightarrow$ 34y = 293 – 110

= 183

$$\Rightarrow y = \frac{183}{34}$$

Question: 8

[5+5=10]

a. A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made at random from the bag without replacements. Find the probability that the first draw yields 3 white balls and the second draw 3 red balls.

Answer:

Total number of items = 30 + 40 = 70

Total number of rusted items = 15 + 20 = 35

Total number of ways of drawing 2 items = $^{70}C_2$

$A \to$ 3 white balls are drawn

$A \to$ 3 red balls are drawn

$$P\left(A \cap \left(\frac{B}{A}\right)\right) = P(A) \times P\left(\frac{B}{A}\right)$$

$$= \frac{^5C_3}{^5C_3} \times \frac{^8C_3}{^{10}C_3}$$

$$= \frac{10 \times 56}{13 \times 22 \times 15 \times 8}$$

$$= \frac{7}{13 \times 33} = \frac{7}{429}$$

b. A box contains 30 bolts and 40 nuts. Half of the bolts and half of the nuts are that either both are rusted. If two items are drawn at random from the box, what is the probability that either both are rusted or both are bolts?

Answer:

$A \to$ both are rusted.

$B \to$ both are bolts.

$$P(A \cup B) = P(A) \times (B) - P(A \cap B)$$

$$= \frac{^{35}C_2}{^5C_3} \times \frac{^{30}C_3}{^{70}C_2} - \frac{^{15}C_2}{^{70}C_2}$$

$$= \frac{595}{2415} + \frac{435}{2415} - \frac{105}{2415}$$

$$= \frac{925}{2415}$$

$$= \frac{155}{483}$$

Question: 9

[5+5=10]

a. Using De Moivre's theorem, find the value of:

$$\left(\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right)^n = \cos n\theta + i \sin n\,\theta, \text{where } i = \sqrt{-1}$$

Answer:

$$\text{L.H.S}\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n$$

$$=\left(\frac{1+2\cos^2\frac{\theta}{2}-1+i\times 2\sin\frac{\theta}{2}\frac{\cos}{2}\theta}{1+2\cos^2\frac{\theta}{2}-1-i\times 2\sin\frac{\theta}{2}\frac{\cos}{2}\theta}\right)$$

$$=\left[\frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)}\right]^n$$

$$\frac{\cos\left(\frac{n\theta}{2}\right)+i\sin\left(\frac{n\theta}{2}\right)}{\cos\left(\frac{n\theta}{2}\right)-i\sin\left(\frac{n\theta}{2}\right)}$$

$$=\frac{\cos\left(\frac{n\theta}{2}\right)+i\sin\left(\frac{n\theta}{2}\right)}{\left[\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right]^{-1}}$$

$$=\left[\cos\left(\frac{n\theta}{2}\right)+i\sin\left(\frac{n\theta}{2}\right)\right]\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)$$

$$=\cos\left(\frac{n\theta}{2}+\frac{n\theta}{2}\right)+i\sin\left(\frac{n\theta}{2}+\frac{n\theta}{2}\right)$$

$= \cos n\theta + i \sin n\,\theta =$ R.H.S(proved)

b. Solve the differential equation:

$$\frac{dy}{dx}-3y\cot x=\sin 2x\text{, given } y = 2 \text{ when } \frac{\pi}{2}$$

Answer:

Given: $\frac{dy}{dx}-3y\cot x=\sin 2x$

On comparing this linear diff. eq. with

$$\frac{dy}{dx}+Py=Q$$

We get P = - 3 cot x, Q = sin 2x

$$\text{I.F.} = e^{\int P\,dx}=e^{\int -3\cot x,\,dx}$$

$$=e^{-3\log\sin x}$$

$$=e^{\log(\sin x)^{-3}}$$

$$=(\sin x)^{-3}=\frac{1}{\sin^3 x}$$

$$\text{y.I.F} = \int Q.\text{I.F}.dx+c$$

$$\frac{y}{\sin^3 x} = \int \sin 2x . \frac{1}{\sin^3 x} dx + c$$

$$\frac{y}{\sin^3 x} = \int \frac{2\sin x \cos x}{\sin^3 x} dx + c$$

$$\Rightarrow \frac{y}{\sin^3 x} = \int 2\operatorname{cosec} x \cot x \, dx + c$$

$$\Rightarrow \frac{y}{\sin^3 x} = \int -2\operatorname{cosec} x \cot x + c$$

$\Rightarrow y = -2 \text{ cosec} . \sin^3 x + c \sin^3 x$

$\Rightarrow y = -2 \sin^2 x + c \sin 3 x$

$y = 2$ when $x = \frac{\pi}{2}$ (given)

$\Rightarrow 2 = -2 + c \times 1$

$\Rightarrow c = 4$

$$\boxed{\therefore\ y = -2\sin^2 x + 4\sin^3 x}$$

Section B (Compulsory) *(Question numbers 10 to 12)*

Question: 10 [5 +5 = 10]

a. For any three vectors $\bar{a}, \bar{b}, \bar{c}$, Prove $\left[\bar{a}-\bar{b}\ \ \bar{b}-\bar{c}\ \ \bar{c}-\bar{a}\right] = 0$

Answer:

We have

L.H.S $\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right]$

$= ((\vec{a}+\vec{b})\times(\vec{b}-\vec{c}).(\vec{c}+\vec{a}))$

$=\left(\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{b}+\vec{b}\times\vec{c}\right).\left(\vec{c}-\vec{a}\right)$

$=\left(\vec{a}\times\vec{b}-\vec{a}\times\vec{c}+\vec{b}\times\vec{c}\right).\left(\vec{c}-\vec{a}\right)$

$=\left(\vec{a}\times\vec{b}+\vec{c}\times\vec{a}+\vec{b}\times\vec{c}\right).\left(\vec{c}-\vec{a}\right)$

$=\left(\vec{a}\times\vec{b}\right).\vec{c}-\left(\vec{a}\times\vec{b}\right).\vec{a}+\left(\vec{c}\times\vec{a}\right).\vec{c}-\left(\vec{c}\times\vec{a}\right).\vec{a}+\left(\vec{b}\times\vec{c}\right).\vec{c}-\left(\vec{b}\times\vec{c}\right).\vec{a}$

$=\left[\left(\vec{a}\,\vec{b}\,\vec{c}\right)\right]-\left[\left(\vec{a}\,\vec{b}\,\vec{a}\right)\right]+\left[\left(\vec{c}\,\vec{a}\,\vec{c}\right)\right]-\left[\left(\vec{c}\,\vec{a}\,\vec{a}\right)\right]+\left[\left(\vec{b}\,\vec{c}\,\vec{c}\right)\right]-\left[\left(\vec{b}\,\vec{c}\,\vec{a}\right)\right]$

$=\left(\vec{a}\,\vec{b}\,\vec{c}\right)-\left(\vec{a}\,\vec{b}\,\vec{c}\right)=0$ [other scalar product having two equal vectors being 0]

b. In any triangle ABC, prove by vector method: $\dfrac{a}{\text{Sin}A}=\dfrac{b}{\text{Sin}B}=\dfrac{c}{\text{Sin}C}$

Answer:

In Δ ABC, Let $\overrightarrow{BC}=\vec{a}, \overrightarrow{CA}=\vec{b}$ and $\overrightarrow{AB}=\vec{c}, a=\left|\overrightarrow{BC}\right|, b=\left|\overrightarrow{CA}\right|$ and $C=\left|\overrightarrow{AB}\right|$

From Δ ABC, $\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{BA}$

$\Rightarrow \overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{BA}=0$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{a}\times 0$

$\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=0$

$\Rightarrow \vec{a}\times\vec{b}=-\vec{a}\times\vec{c}=\vec{c}\times\vec{a}$.........(i)

Similarly $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$ ....(ii)

eq.(i) and (ii) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|=\left|\vec{b}\times\vec{c}\right|=\left|\vec{c}\times\vec{a}\right|$

$\Rightarrow$ a b sin $(\pi$-C$)$= bc sin $(\pi$-A$)$= ca sin $(\pi$-B$)$

$\Rightarrow$ ab sin C = bc sin A = ca sin B

$\Rightarrow \dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

$\Rightarrow \dfrac{\sin c}{C}=\dfrac{\sin\ A}{a}=\dfrac{\sin B}{b}$

Question: 11

[5+5=10]

a. Find the shortest distance between the lines:

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{5-z}{5}$$

Answer:

Vector form $\vec{r}=8\hat{i}-9\hat{j}+10\hat{k}+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$

And $\vec{r}=15\hat{i}-29\hat{j}+5\hat{k}+\lambda\left(3\hat{i}+8\hat{j}+7\hat{k}\right)$

Comparing these equations with

$\vec{r}=\vec{a_1}+\lambda\vec{b_1} \text{ and } \vec{r}=\vec{a_2}+\mu\vec{b_2}$

$\vec{a_1}=8\hat{i}-9\hat{j}+10\hat{k},\ \vec{a_2}=15\hat{i}-29\hat{j}+5\hat{k}$

$\vec{b_1}=3\hat{i}-16\hat{j}+7\hat{k},\ \vec{b_2}=3\hat{i}+8\hat{j}-5\hat{k}$

$\vec{a_2}-\vec{a_1}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)-\left(8\hat{i}-9\hat{j}+10\hat{k}\right)$

$=7\hat{i}+38\hat{j}-5\hat{k}$

$$\text{Now, } \vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}$$

$=(80-56)\hat{i}-(-15-21)\hat{j}+(24+48)\hat{k}$

$=24\hat{i}+36\hat{j}+72\hat{k}$

$\vec{b_1}\times\vec{b_2}=\sqrt{(24)^2+(36)^2+(72)^2}$

$=\sqrt{576+1296+5184}=\sqrt{7056}=84$

$$\text{S.D} = \left|\frac{\left(\vec{a_2}-\vec{b_1}\right).\left(\vec{b_1}-\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|$$

$$=\left|\frac{\left(7\hat{i}+38\hat{j}-5\hat{k}\right).\left(24\hat{i}+36\hat{j}+72\hat{k}\right)}{84}\right|$$

$$=\left|\frac{168+1368-360}{84}\right|=\left|\frac{1176}{84}\right|$$

= 14 units.

b. Find the equation of the plane passing through the lines of intersection of the planes x+2y+3z – 5 = 0 and 3x – 2y-z + 1 = 0 and cutting off equal intercepts on the x and z axes.

Answer:

Equation of the plane passing through the lines of intersection of the planes x+2y+3z – 5 = 0 and 3x – 2y-z + 1 = 0 is

$(x+2y+3z-5)+\lambda(3x-2y-z+1=0)$ .....(i)

$\Rightarrow (1+3\lambda)x+(2-2\lambda)y\ (3-\lambda)z+\lambda-5=0$

$\Rightarrow (1+3\lambda)x+(2-2\lambda)y\ (3-\lambda)z=5-\lambda$

$$\frac{x}{\left(\frac{5-\lambda}{1+3\lambda}\right)}+\frac{y}{\left(\frac{5-\lambda}{1+3\lambda}\right)}+\frac{z}{\left(\frac{5-\lambda}{3-\lambda}\right)}=1$$

Comparing it with $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$$a=\frac{5-\lambda}{1+3\lambda}, b=\frac{5-\lambda}{1+3\lambda}, c=\frac{5-\lambda}{3-\lambda}$$

$$\Rightarrow \left(\frac{5-\lambda}{1+3\lambda}\right)=\left(\frac{5-\lambda}{3-\lambda}\right)$$

$$\Rightarrow 1+3\lambda=3-\lambda$$

$$\Rightarrow 4\lambda=2$$

$$\Rightarrow \lambda=\frac{1}{2}$$

$$(x+2y+3z-5)+\frac{1}{2}(3x-2y-z+1)=0$$

$\Rightarrow$ 2x + 4y + 6z – 10 + 3x-2y-z+1=0

5x+2y+5z-9=0

Question: 12 [5+5=10]

a. In a class of 75 students, 15 are above average, 45 are average and the rest below average achievers. The probability that an above average achieving student fails is 0.005, that an average achieving student failing is 0.15. if a student is known to have passed, what is the probability that he is a below average achiever?

Answer:

$E_1 \rightarrow$ Above average

$E_2 \rightarrow$ Average

$E_3 \rightarrow$ Below average

$$P(E_1)=\frac{15}{75}=\frac{1}{5}, P(E_2)=\frac{45}{75}=\frac{3}{5}, P(E_3)=\frac{15}{75}=\frac{1}{5}$$

$$P\left(\frac{A}{E_1}\right)=1-\frac{5}{1000}=\frac{995}{1000}=\frac{199}{200}$$

$$P\left(\frac{A}{E_2}\right)=1-\frac{5}{100}=\frac{95}{100}$$

$$P\left(\frac{A}{E_3}\right)=1-\frac{15}{100}=\frac{85}{100}$$

$$P\left(\frac{E_3}{A}\right)=\frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{5}\times\frac{85}{100}}{\frac{1}{5}\times\frac{199}{200}+\frac{3}{5}\times\frac{95}{100}+\frac{1}{5}\times\frac{85}{100}}$$

$$= \frac{\frac{85}{500}}{\frac{199}{5000} + \frac{285}{500} + \frac{85}{500}}$$

$$= \frac{\frac{85}{500}}{\frac{199 + 285 + 85}{1000}}$$

$$= \frac{85}{500} \times \frac{1000}{939} = \frac{170}{939}$$

b. The probability that a bulb produced by a factory will fuse in 100 days of use is 0.05. find the probability that out of 5 such bulbs, after 100 days of use:

i. None fuse
ii. Not more than one fuses
iii. More than one fuses.
iv. At least one fuses.

Answer:

Let P = Probability that a bulb will fuse $= 0.05 = \frac{5}{100} = \frac{1}{20}$

$q = 1 - \frac{1}{20} = \frac{19}{20}$

Let X = no. of bulbs that will fuse after 100 days of use

Here n = 5

$P(X = r) = {}^{n}CrP^{r}q^{n-r}$

i. $P(X = 0) = {}^{5}C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{(5-0)} = \left(\frac{19}{20}\right)^5$

ii. P(X= 0 or X = 1) = P(X = 0) + P(X = 1)

$$= {}^{5}C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{(5-0)} + {}^{5}C_1\left(\frac{1}{20}\right)^1\left(\frac{19}{20}\right)^{(5-1)}$$

$$= \left(\frac{19}{20}\right)^{(5)} + \frac{5}{20} \times \left(\frac{19}{20}\right)^{(4)}$$

$$= \left(\frac{19}{20}\right)^{(4)}\left[\frac{19}{20} + \frac{1}{4}\right] = \left(\frac{19}{20}\right)^{(4)} \times \frac{24}{20}$$

$$= \frac{6}{5} \times \left(\frac{19}{20}\right)^{(4)}$$

$$= 1 - \frac{6}{5}\left(\frac{19}{20}\right)^{(4)}$$

$$= 1 - \left(\frac{19}{20}\right)^5$$

Section C(Statistics (Question numbers 13 to 15)

Question 13 [5+5=10]

a. Two tailors P and Q earn Rs. 150 and Rs. 200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost?

Answer:

Let tailor P work for x days and tailor Q works for y days.

According to problem

$6x + 10y \geq 60$

$\Rightarrow 3x + 5y \geq 30$ [For shirts]

$\Rightarrow 4x + 45y \geq 32$

$\Rightarrow x + y \geq 8$ [For trousers]

Cost (Z) = 150x + 200y

$x \geq 0, y \geq 0$

We need to minimize (Z) subject to constraitns (1), (2) and (4).

Now we draw the lines

$3x + 5y = 30, x+y = 8$

$3x+5y = 30$

$x + y = 8$

x	0	10
y	6	0

x	0	8
y	8	0

These lines meet at (5,3). The Feasible region has been shaded and it is an unbounded region with vertices at A, B and C.

PICTURE

At (A(10,0) the cost (Z) = 150 x 10 + 200 x 0 = Rs. 1500.

At (B(5,3) the cost (Z) = 150x + 200y = Rs. 150 x 5 + 200 x 3 = 750+ 600 = Rs. 1600

At (C(0,8) the cost (Z) = 150 x 0 + 200 x 8 = Rs. 1600

At B(5,3) , Z is minimum. Therefore P should for 5 days and Q should work 3 days.

b. A machine costs Rs. 97000 and its effective life is estimated to be 12 years. If scrap realizes Rs. 2,000 only, what amount should be retained out of profits at the end of each year to accumulate at compound interest of 5% per annum in order to buy a new machine after 12 years ? (use $1.05^{12} = 1.769$)

Answer:

Let a be the annual instalment. It is obvious that the amount of the annuity to continue for 12 years i.e, the balance amount to be retained.

= Rs. 97000 – Rs 2000 = Rs. 95000

$$A = 95000, i = \frac{5}{100} = 0.05$$

$$A = \frac{a}{i}\left[(1+i)^n - 1\right]$$

$$95000 = \frac{a}{0.05}\left[(1+0.05)^{12} - 1\right]$$

$$= \frac{a}{0.05}\left[(1.05)^{12} - 1\right]$$

$$= \frac{a}{0.05}[1.769 - 1]$$

$$= \frac{a}{0.05} \times 0.769$$

$$a = \frac{95000 \times 0.05}{0.769} = \frac{4750}{0.769}$$

Question 14 [5+5=10]

a. A bill of Rs. 1000 drawn on 7th May, 2011 for six months was discounted on 29th August , 2011 for cash payment of Rs. 988. Find the rate of interest charged by the bank. [5]

Answer:
Here bill value (A) = Rs. 1000.
Bill is accepted on 7th May;
Bill is due on 7th November
Bill is discounted on 29th August
Let Rate of interest be r% per annum
B.D = Bill value – discounted value
= 1000 – 988
= Rs. 12
B.D = An i

August	02
September	30
October	31
November	07
Days of grace	03
	73 days

$$\Rightarrow 12 = 1000 \times \frac{73}{365} \times \frac{r}{100}$$

$\Rightarrow$ 12 = 2r
$\Rightarrow$ r = 6% per annum

b. If total cost function is given by C = a + bx+cx2, where x is the quantity of output. Show that: [5]

$$\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$$

Answer:
Here $C = a + bx + cx^2$

$$AC = \frac{C}{x} = \frac{1}{x}\left(a + bx + cx^2\right)$$

$$= \frac{a}{x} + b + cx$$

$$MC = \frac{dC}{dx} = \frac{d}{dx}\left(a + bx + cx^2\right)$$

$$= b + 2cx$$

$$\frac{d}{dx}(AC) = \frac{d}{dx}\left(a + bx + cx^2\right)$$

$$= -\frac{a}{x^2} + c$$

$$\frac{1}{x}(MC - AC) = \frac{1}{x}\left(b + 2cx - \frac{a}{x} - b - cx\right)$$

$$\frac{1}{x}(MC - AC) = \frac{1}{x}\left(cx - \frac{a}{x}\right)$$

$$\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$$

Question 15

[5+5=10]

a. Find the consumer price index number for the year 2010 using year 2000 as the base year by using method of weighted aggregates:

Commodity	A	B	C	D	E
Year 2000 price (in Rs. Per unit)	16	40	0.50	5.12	2
Year 2010 price (in Rs. Per unit)	20	60	0.50	6.25	1.50
Weights	40	25	5	20	10

Answer:

Commodity	Price in year 2000 (P_0)	Price in year 2000 (P_1)	Weights(W)	P_1W	P_0W
A	16	20	40	800	640
B	40	60	25	1500	1000
C	0.50	0.50	5	2.50	2.50
D	5.12	6.25	20	125	102.40
E	2	1.50	10	15	20
				ΣP_1w= 2442.5	ΣP_2w= 1764.90

$$P_{01}\frac{\sum P_1 w}{\sum P_2 w}\times 100$$

$$= \frac{2442.5}{1764.90}\times 100$$

Index Number = $= 138.4$

b. Calculate the 5 yearly moving average of the number of students in a college from the following data and plot them on a graph paper:

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Number	332	317	357	392	402	405	510	427	405	438

Calculate the four monthly moving averages and plot these and the original data on a graph sheet.

Answer:

Computation of Five Yearly Moving Average

Year	No. of students	5 yearly moving total	5 yearly moving average
1981	332	-	-
1982	317	-	-
1983	357	1800	360
1984	392	1873	374.6= 375
1984	402	2066	413.2=413
1985	405	2136	427.2=427
1986	510	2149	429.8=430
1987	427	2185	437
1988	405		
1989	438		
1990			